An Alternative Method to α–λ for Evaluating the Relative Accuracy of Prognostic Information

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Abstract

Valuating the accuracy of information produced by a set of prediction algorithms is a prime question in prognostics. Equally important is a quantification of the efficiency of convergence from an initial estimate error to an estimate error that falls within an α-margin of accuracy. This paper presents an overview of qualifying significant prognostic information related to Remaining Useful Life (RUL), State of Health (SoH), Prognostic Distance (PD), and Prognostic Horizon (PH). Models and metrics related to that set of information are developed and presented. A NASA-identified accuracy problem with the α–λ method for evaluating relative accuracy of RUL estimates is presented and explained, and an alternative method is presented as a solution. The alternative method comprises a figure of merit, Convergence Efficiency, to evaluate the convergence of estimates of PH to within an α-margin of accuracy. Estimates of PH include estimates of RUL, PD, and end-of-life (EOL) and are produced each time a new input data point is processed. The paper ends with a conclusion section.

1. Introduction

A Prognostic Health Management (PHM) system comprises frameworks (CAVE3 2015, Goodman, Hofmeister, and Szidarovszky 2019, Kumar and Pecht 2010) that may be realized as shown in Figure 1. The first stage of prognostic monitoring is the sensing section, observing one or more nodes of a system and collecting Condition-based Data (CBD) containing noise and features useful for determining the health of a system. : Figure 2 is an example of CBD comprising noise and features: a feature not correlated to a failure mode of interest becomes noise.

The second stage performs signal processing serving data processing and feature extraction that includes, for example, data and domain transforms. The output of that stage is feature data (FD) that is also known as condition indicators and/or precursors to failure. (Goodman, Hofmeister, and Szidarovszky (2019), Hofmeister, Goodman, and Szidarovszky (2017, 2018), IEEE Draft Standard (2018)).

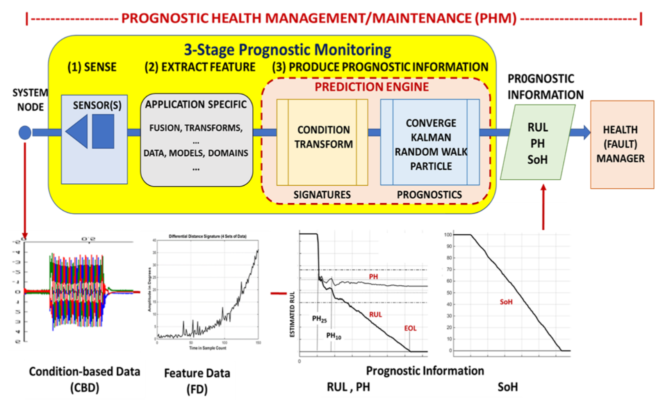

Figure 1. Diagram of a PHM system with a 3-stage monitoring system: (1) sensing, (2) data processing, and (3) prognosing.

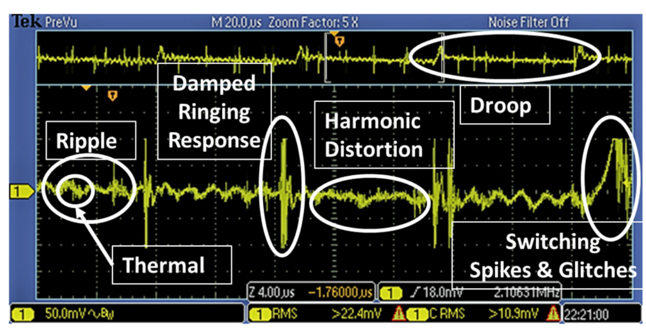

Figure 2. Example of CBD comprising noise and features at the output of a switch-mode power supply.

One feature shown in Figure 2, the damped-ringing response, also contains additional features as shown in Figure 3.

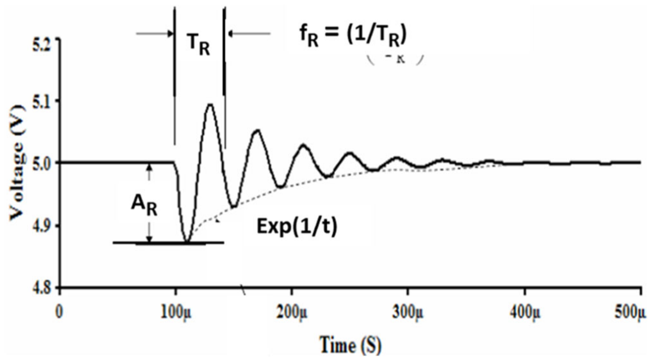

Figure 3. Damped-ringing response with three features: amplitude, resonant frequency, and exponential decay.

1.1. Features and Signatures

Referring back to Figure 2 and Figure 3, in the presence of degradation, FD forms a signature that can be modeled by various approaches such as Physics of Failure (PoF) and Failure Mode Effects Analysis (FMEA) as a function of a change in value (dP) of a parameter (P). For example, the filtering capacitors in the output of a power supply are prone to failure. Modeling includes the following (Judkins and Hofmeister, 2007):

- Multiple features (CBD) and noise (N)

$$V = CBD_1 + CBD_2 + \cdots CBD_n + N_1 + N_2 + N_m \quad (1)$$

- Damped-ringing response (a CBD feature)

$$V_0 = V_{DC} + A_R\{exp(-t/\lambda)\}\{cos(\omega t + \phi)\} \quad (2)$$

- Natural resonant frequency

$$\omega_0 = \sqrt{A_R + 1}\ (1/\sqrt{LC}) \quad (3)$$

- Circuit quality

$$Q = \sqrt{A_R + 1}\ (1/R)(\sqrt{C/L}) \quad (4)$$

- Measurable frequency

$$\omega = \omega_0\sqrt{1 - 1/(4\,Q^2)} \quad (5)$$

- Simplify frequency model using substitution, assuming a high value of Q (≥ 10), and using insignificant variations in the values of A, R, and L compared to changes in value of C due to degradation:

$$\omega \approx \omega_0\sqrt{(\,C0/(C0 - dC))} \quad (6)$$

  C0 is a nominal value of C and *dC* represents a change in value of C.

1.2. Signature Transforms

A set of feature values $\{FD_i\}$ forms a characteristic curve: a signature such as that shown in Figure 4.

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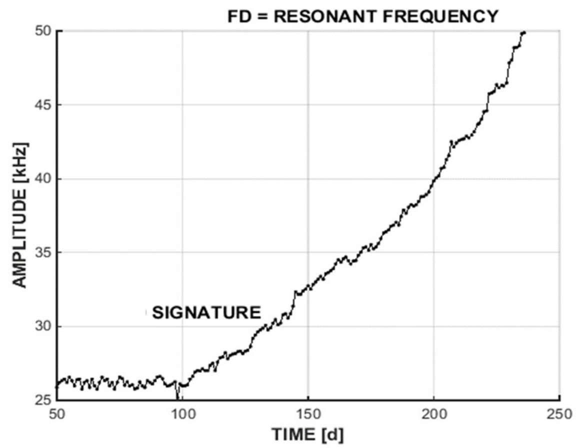

Figure 4. Example of a signature: FD is the resonant frequency that changes as capacitance is reduced in the output filter of a switch-mode power supply.

- Feature data

$$FD = \omega/2\pi \quad (7)$$

$$FD_0 = \omega_0/2\pi \quad (8)$$

1.2.1. Functional Failure Progression (FFP) Signature

Functional failure is defined as a state-of-health and operation wherein a prognostic target is no longer operating within specifications. The data points comprising an FFP signature are calculated as follows:

- Let NM represent a noise margin to mitigate noise, let FD_i represent the FD value of an i-th sample, and let FD_{res} represent a value to canonicalize data to a scale such that 1.0 represents approximately a doubling in value

$$FFP_i = (FD_i - FD_0 - NM)/FD_{res} \quad (9)$$

- With dP being a change in value of a parameter P, representing C0. If FD_o represents the nominal value of the resonant frequency of a damped-ringing response, and FD, the resonant frequency of a damped-ringing response in the presence of degradation, then:

$$FFP_i = f(FD_i\,, FD_0)g(dP, P) \quad (10)$$

Figure 5 is an example of an FFP signature.

1.2.2. Functional Failure Signature (FFS)

Dividing FFP signature data, or a derivative signature of it, by a value that represents a failure threshold and multiplying it by 100, creates a Functional Failure Signature (FFS):

- Let FL = an FFP value at which a prognostic target is no longer able to operate within specifications (functional failure)

$$FFS_i = 100\,FFP_i/FL \quad (11)$$

An FFS has the following properties that are very amenable in prognostics: (1) when FFS ≤ 0, there is no degradation; (2) when FSS ≥ 100, functional failure has occurred; (3) otherwise, the prognostic target (a power supply for example) is operating in a degraded state: see Figure 6.

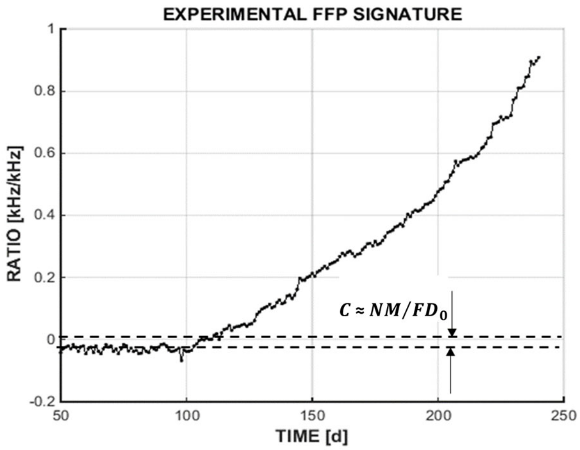

Figure 5. Example FFP signature.

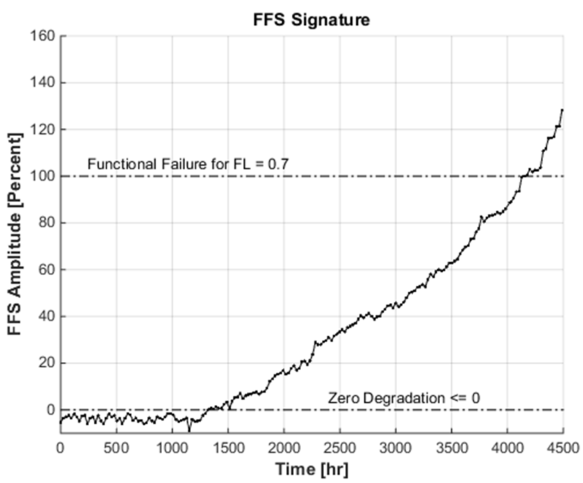

Figure 6. Example FFS.

1.2.3. Degradation Progression Signature (DPS)

EQ. (10) can be solved to produce a Degradation Progression Signature (DPS) which is defined as

$$DPS_i = dP_i/P = h(FFP_i) \qquad (12)$$

In the case of our example power supply, it can be shown that

$$FFP_i = (FD_i/FD_0)([1/(1 - dP_i/P_0)]^n - 1) + c \quad (13)$$

$$DPS_i = 1 - 1/(FFP_i + 1)^{1/n} + d \qquad (14)$$

Where n = ½, c is a constant, d is a constant

Since a DPS is essentially a derivative of an FFP signature, an FFP to DPS transform linearizes curvilinear signatures. See Figure 7 for an example DPS and compare that to Figure 5.

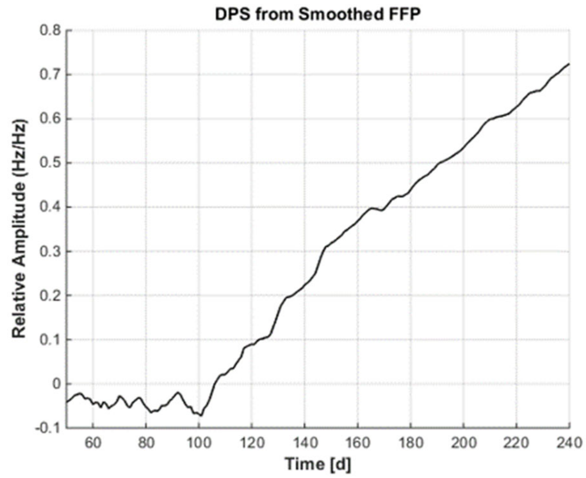

Figure 7. Example DPS from the FFP signature data shown in Figure 5 after data smoothing.

1.2.4. Transform DPS Data to FFS Data

DPS data is further transformed to FFS data by performing the following as each DPS data point is created:

- Transform the FFP-based FL to a DPS-based FL

$$FL_{DPS} = 1 - 1/(FL_{FFP} + 1)^{1/n} \qquad (15)$$

- Transform each DPS data point to an FFS data point

$$FFS_i = 100\, DPS_i/FL_{DPS} \qquad (16)$$

The FFS for the example we have been using is shown in Figure 8: compare that to Figure 6 and note the following: the FFS from a DPS is more linear than an FFS from the FFP signature, and data smoothing of an FFP signature results in a smoothed FFS.

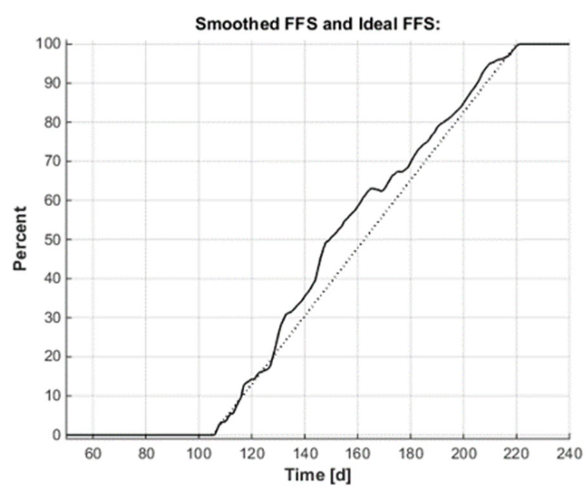

Figure 8. Example FFS from the DPS shown in Figure 7.

1.3. Prognostic Information

1.3.1. Prognostic Information and Ideality

Ideally, the prognostic information produced by a prediction engine will be ideal, regardless of linearity of the input data: see the example plots in Figure 9 and Figure 10. The PA (Prognostic Accuracy) represents a condition where every RUL estimate, which is used to estimate PH, is exactly correct.

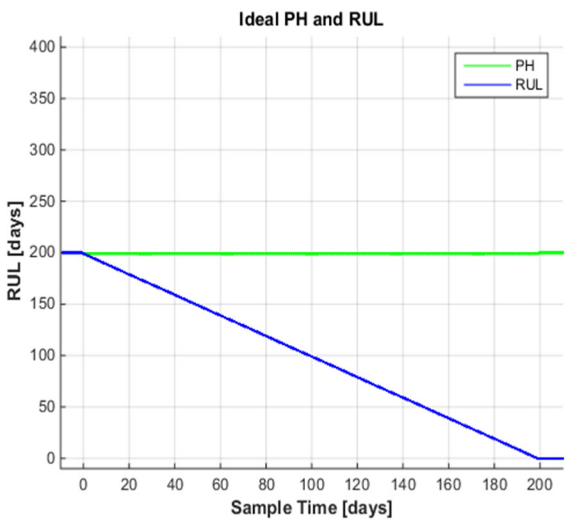

Figure 9. Example of ideal RUL and PH plots.

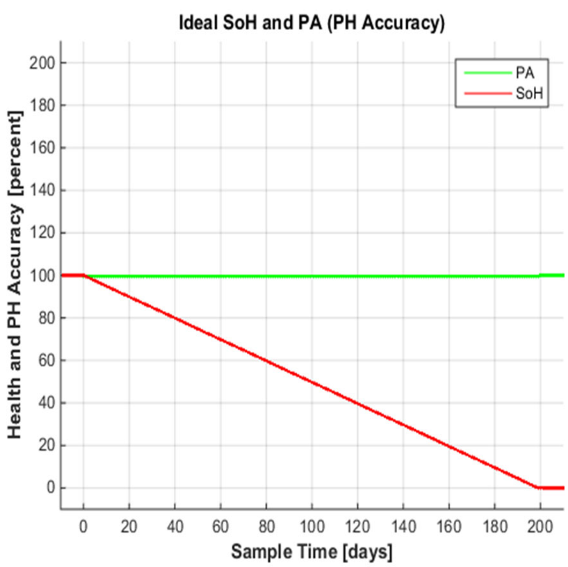

Figure 10. Example SoH plot (red).

Such ideality is impossible, if for no other reason that it is impossible to know exactly how long it will take for every instantiation of a prognostic target to fail prior to detection of degradation and subsequent functional failure. The prediction engine used to create prediction information uses a system specified Predicted Initial Time-to-Functional Failure (PITTFF) as an initial RUL value: that value will either be higher or lower than the actual time it takes to progress from the onset of degradation to functional failure. At best, the objective is to produce prognostic information that approximates pseudo-ideal such as that shown in Figure 11 and Figure 12.

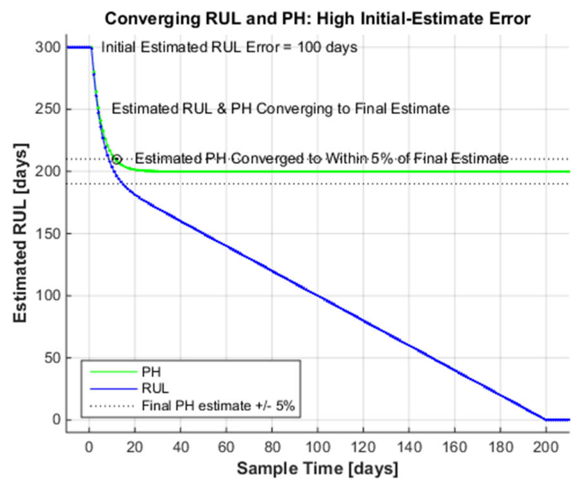

Figure 11. Example pseudo-ideal RUL and PH plots, given a higher than actual value of PITTFF.

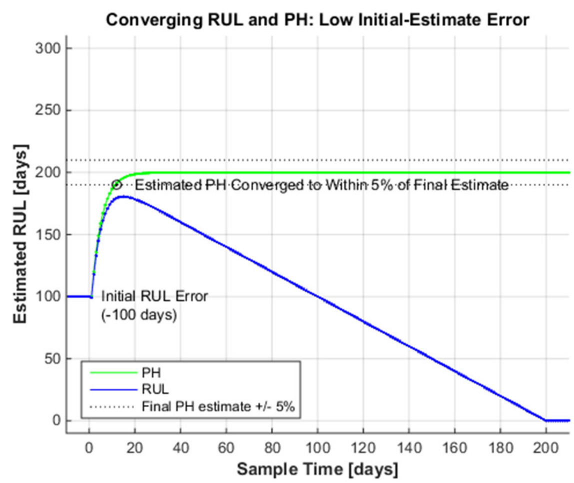

Figure 12. Example pseudo-ideal RUL and PH plots given a lower than actual value of PITTF.

1.3.2. Realistic Prognostic Information

Real data is not ideal. In addition to noise, there are variances from ideal that are not related to degradation such as, for example, sampling periods, quantization errors because of analog-to-digital and digital-to-analog data conversions, computational rounding, and the effects of feedback: see Figure 13. At best, then, prognostic information is going to look like either the top (solid) or the bottom (dashed) plots seen in Figure 14.

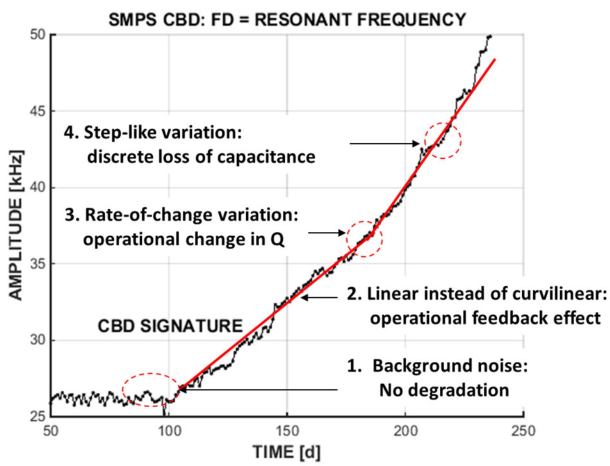

Figure 13. Examples of non-ideality in a signature.

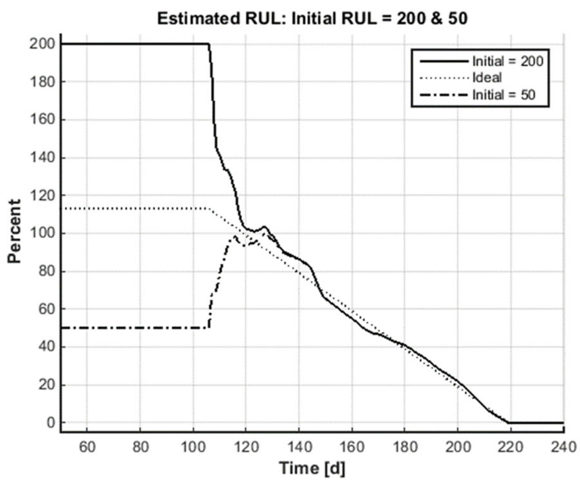

Figure 14. Example plots: realistic RUL estimates are noisy and not straight lines (top and bottom plots) rather than the straight-lines (middle plot).

1.4. Prognostic Information and Terminology

Referring to Figure 15, there are two prime events: one is when degradation is detected (BD) and the other one when functional failure occurs (EOL). BD occurs after the true time of onset of degradation because sampling (TS) is periodic. EOL can occur either before or after true functional failure because data smoothing is typically employed and that tends to shift signatures downward. The prognostic information of interest are RUL, PH, and PD

1.4.1. RUL

RUL estimates are defined as the time between the time of sampling and the estimated EOL for that sample

$$RUL_i = EOL_i - TS_i \tag{17}$$

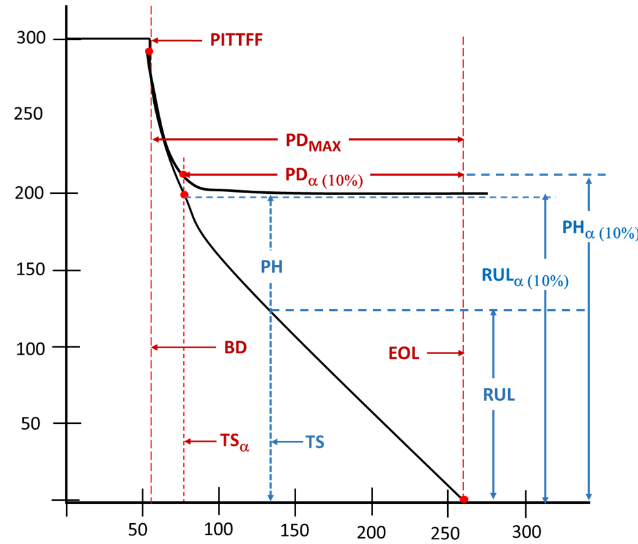

Figure 15. RUL and PH plots and related terms.

1.4.2. PH and PD

In this paper, definition of PH is defined as the time since degradation was detected plus the RUL estimate for the sampled data

$$PH_i = (TS_i - BD) + RUL_i \tag{18}$$

Ideally, all values of PH_i would be equal to $EOL - BD$,

PD is defined as the distance in time between the i-th estimate of EOL and the i-th time of a sample

$$PD_i = EOL_i - TS_i \tag{19}$$

$$PD_{MAX} = EOL - BD \tag{20}$$

2. RELATIVE ACCURACY AND CONVERGENCE EFFICIENCY

Relative accuracy means how accurate is a measurement or calculation relative to something.

2.1. α–λ Method of Evaluating RA

Saxena, Celaya, Saha, Saha, and Goebel (2009) defined relative accuracy as the accuracy of RUL_i relative to the true value of RUL at that time and named the method α–λ,

$$RUL_{TRUE} = EOL_{TRUE} - TS_i \tag{21}$$

$$\alpha\text{–}\lambda = 100(RUL_i - RUL_{TRUE})/RUL_{TRUE} \tag{22}$$

A problem with EQ. (22) is the denominator, RUL_{TRUE}, decreases in value over time. For example, suppose the time of sampling is 10.2 sampling periods from the time of EOL_{TRUE}. Further suppose a prediction engine estimates RUL as being 10.3 sampling periods (99% accurate). Ten sampling periods later, RUL_{TRUE} is reduced to 0.3 and RUL_i

is reduced to 0.2 and from EQ. (22), α–λ is calculated as 50% a very large accuracy error (see Figure 16).

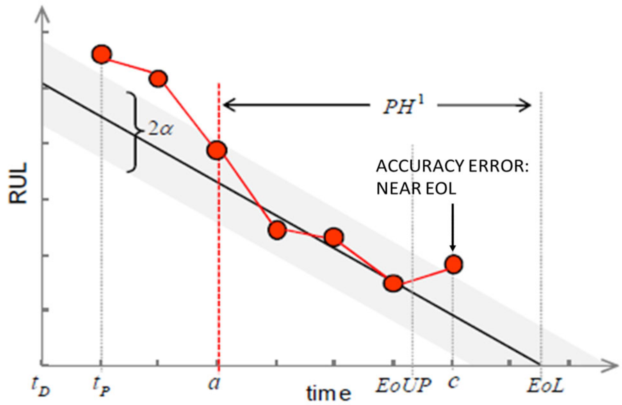

Figure 16. Relative accuracy plot (after Saxena et al. 2009.

The accuracy error is unavoidable as one should not calculate a value with a greater degree of precision than the precision of the measurement. In this case, the precision is one sampling period: hence one should not produce a value of 10.2 for RUL but round it to 10.

2.2. Accuracy, Precision, and Sampling

Accuracy and precision requirements and sampling must be consistent.

For example, suppose the PHM requirements are the following: (1) notification of failure must be at least 72 hours in advance with an accuracy of $\pm$ one hour, and (2) 12 hours before failure, predicted time of failure must be accurate to within 5%. Those requirements lead to a sampling rate of at least 4 times per hour:

5% of 12 hours equals 0.6 hours

20 periods of 0.6 hours in 12 hours

40 samples in 12 hours (absent significant noise)

Minimum sampling rate of 3.33 times per hour

Increase to 4 times per hour

2.3. α-Margin of Accuracy

An alternative to α–λ is to use the following approach:

- Use EQ. (18) to calculate values of PH_i
- Calculate an estimate for PD_{MAX}

$$PD_{MAX(i)} = max(EOL_{i-5}^{i} - BD) \qquad (23)$$

- Calculate a Margin of Accuracy

$$\alpha = 100(PH_i/PD_{MAX(i)} - 1) \qquad (24)$$

- When α is within a specified value, then

$PD_\alpha = PD_i$ at the time when PH_i is at first within criterium.

2.4. Convergence Efficiency Method of Evaluating RA

The proposed approach defines and uses a Convergence Figure of Merit (χ_α) that is defined as follows:

- Let α be a specified margin of accuracy such as 25%, 10%, or 5%,
- Let TS_α be the time of sampling when PH_i is within the specified α-margin of accuracy for all subsequent samples.
- Let PH_α be a value of PH_i equal to $PD_{MAX}(1 \pm \alpha/100)$,
- Let PD_α be the distance when PH_α criterium is met for all subsequent times of samples (TS_i)

  Then the Convergence Figure of Merit is the value of χ when all subsequent values of PH_i are within the margin of accuracy defined by PH_α,

$$PD_\alpha = EOL_\alpha - TS_\alpha \qquad (25)$$

$$\chi_\alpha = (PD_\alpha/PD_{MAX}) \qquad (26)$$

2.4.1. Use as a Metric for Evaluation and Verifications

Convergence is a useful performance metric to determine whether a prognostic system and its set of operating specifics meets requirements regarding performance. A typical set of performance requirements might be stated in the following manner:

- The prognostic system must detect degradation at least 2,500 hours before failure.

$$PD_{MAX} = \geq 2{,}500 \ hours$$

- The prognostic system must predict failure at least three quarters of the prognostic distance before failure with at least 25% accuracy

$$\chi_\alpha = \chi_{25} \geq 0.75$$

- The prognostic system must predict failure at least one-half of the prognostic distance before failure with at least 10% accuracy

$$\chi_\alpha = \chi_{10} \geq 0.50$$

2.4.2. Example 1, Failed to Meet Requirements

Referring to Figure 17, first criterium, PD_{MAX}, is met, the second criterium, χ_{25}, is met, and the third criterium, χ_{10}, is not met.

2.4.3. Example 2, Met Requirements

Referring to Figure 18, the first two criteria (PD_{MAX} and χ_{25}) are well within requirements and the third criterium, χ_{10}, is barely within specifications.

2.4.4. Example 3, Far Exceeded Requirement

As seen in Figure 19, all criteria are easily met.

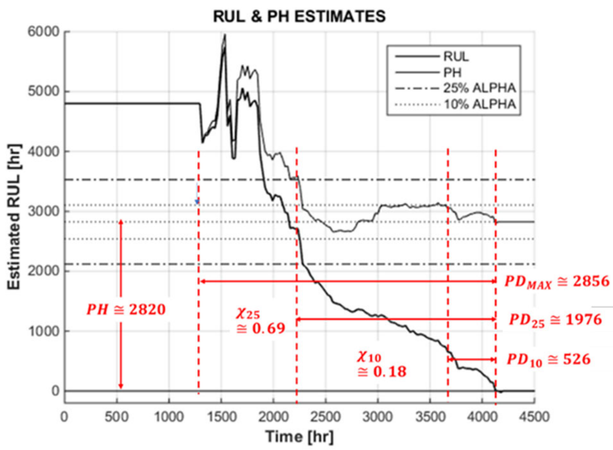

Figure 17. Example RUL and PH plots: failure to meet accuracy requirements

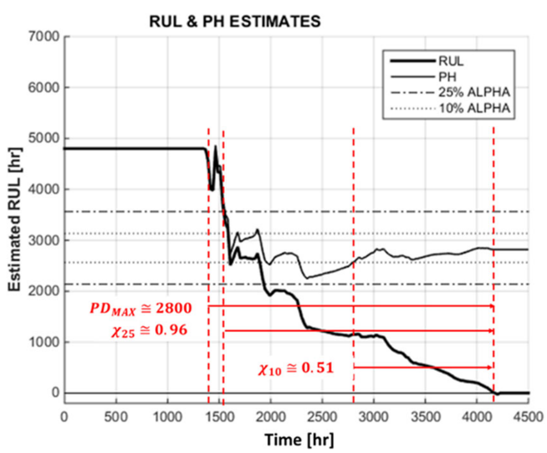

Figure 18. Example RUL and PH plots all criteria exceed requirements

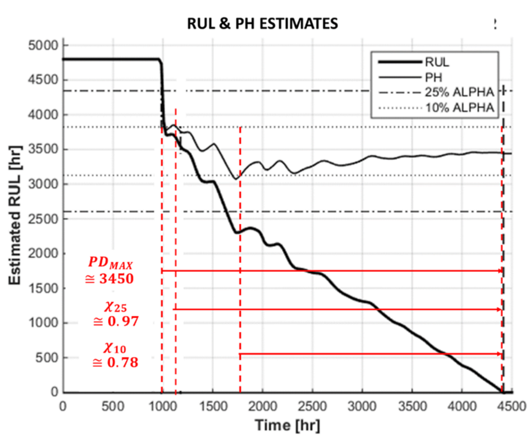

Figure 19. Example RUL and PH plots: far exceeds requirement for $\alpha = 10\%$

3. CONCLUSION

This paper provides an explanation for inaccuracies using an α–λ method, which is relative to RUL, an ever-decreasing denominator. An alternative method, α-margin of accuracy, which is PH relative to a fixed value parameter, PD, in the denominator: refer back to EQ. (26) and compare to EQ. (22). A figure of merit, Convergence Efficiency, was developed and examples were used to illustrate the use of Convergence Efficiency: EQ. (25) and EQ. (26).

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