# Degradation Modeling from Condition-based Data to Functional Failure Signature Data

James P. Hofmeister
Ridgetop Group, Inc.
3580 West Ina Road
Tucson, AZ 86741
520-742-3300x107
hoffy@ridgetopgroup.com

Douglas L. Goodman
Ridgetop Group, Inc.
3580 West Ina Road
Tucson, AZ 86741
520-742-3300
doug@ridgetopgroup.com

Ferenc Szidarovszky
Ridgetop Group, Inc.
3580 West Ina Road
Tucson, AZ 86741
520-742-3300
szidarka@gmail.com

*Abstract*—**This paper presents approaches to degradation modeling starting with condition-based data (CBD) and progressing to functional-failure signature (FFS) data: FFS data forms a transfer curve that is very amenable to processing by prediction algorithms in support of a Prognostic Health Monitoring (PHM) system. The approach uses degradation signal models that are previously developed, validated, and presented: for example, an MFPT 2018 paper "Degradation Signal Modeling." Degradation signal modeling transforms curvilinear, CBD-based signature data into signature data that is much more linearized, which increases the accuracy of prediction information such as remaining useful life (RUL) and state of health (SoH). The focus of this paper is transforming CBD signatures into fault-to-failure progression (FFP) signatures, degradation-progression signatures (DPS), and then into FFS [1]-[3].**

## 1. Introduction

A Prognostic Health Monitoring (PHM) system uses sensors to acquire data and processes that data for the purpose of detecting condition indicators (precursors to failure) and using that data to prognose the state-of-health of a system by producing prognostic information to include, for example, the state-of-health (SoH) and remaining useful life (RUL) of the system. Important goals include maximizing the time between when a less than 100% healthy state is first detected and the time when functional failure occurs. Functional failure is defined to occur at the level of degradation at which a device, component, or assembly is no longer operating within specifications. Condition-based data (CBD comprises features and noise: features being anything that can be used for diagnostic and/or prognostic purposes and noise is everything else. The left side of Figure 1 is an oscilloscope capture of the output of a switch-mode power supply: feature data (FD) includes ripple voltage, voltage droop (see the upper trace), and what is often termed to be spikes and glitches, such as that circled in red. Typically, though, those spikes and glitches are what is called a damped-ringing response (right side of Figure 1).

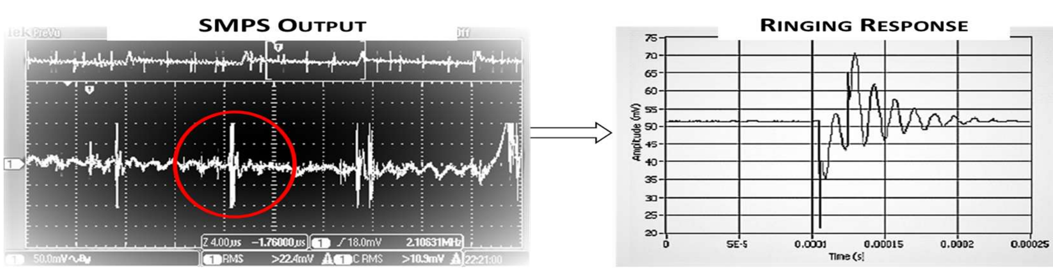


**Figure 1. SMPS output voltage (left) and extracted damped-ringing response (right)**

A damped-ringing response can be modeled as comprising many features such as a DC voltage, $V_{DC}$; a response amplitude, $A_R$; a dampening time constant, $\tau$; a frequency, $\omega$; and a phase shift, $\phi$ [4],[5]:

$$V_0 = V_{\mathrm{DC}} + A_R\{\exp(-t/\tau)\}\{\cos(\boldsymbol{\omega} t + \phi)\} \tag{1}$$

PoF analysis shows that as the filter capacitance of an SMPS degrades, the resonant frequency changes:

$$\boldsymbol{\omega} \approx \omega_0\sqrt{C_0/(C_0 - \Delta C)} \tag{2}$$

Then from $f = \omega/2\pi = FD$,

$$FD = FD_0\sqrt{C_0/(C_0 - \Delta C)} \tag{3}$$

Substituting $P_0$ (nominal parameter value) for $C_0$ and $dP$ (change in parameter) for $\Delta C$, the radical can be replaced by $g(dP, P_0)$:

$$FD = FD_0\, \mathrm{g}(dP, P_0) \tag{4}$$

Which is interpreted to mean the following: the measurement from a sensor is the nominal value of a parameter times a function that changes in correlation to degradation where $dP$ is the change in value of that parameter. A fault-to-failure progression (FFP) data point is defined to be the following:

$$FFP_i = (FD_i - FD_0 - NM)/FD_0 \tag{5}$$

Where $NM$ is a constant value chosen to mitigate the effects of noise so that an FFP data point > 0 indicative of a degraded state. By collecting terms, and letting $(-FD_0 - NM)/FD_0 = -\mathrm{C}$ (a constant), Eq. (5) becomes

$$FFP_i = FD_i/FD_0 - \mathrm{C}$$

And by substitution using Eq. (4)

$$FFP_i = \mathrm{g}(dP, P_0) - \mathrm{C} \tag{7}$$

***Fault-to-Failure Progression (FFP) Signature:***

A set of FFP data points forms a collection of normalized, dimensionless data obtained by transforming FD points starting with a particular point, $m$, and ending at a final point, $m$+$n$, that forms a signature (see Figure 2):

$$\boldsymbol{FFP} = \{FFP_{m,}\, FFP_{m+1}, \ldots\, FFP_{m+n-1}, FFP_{m+n}\} \tag{8}$$

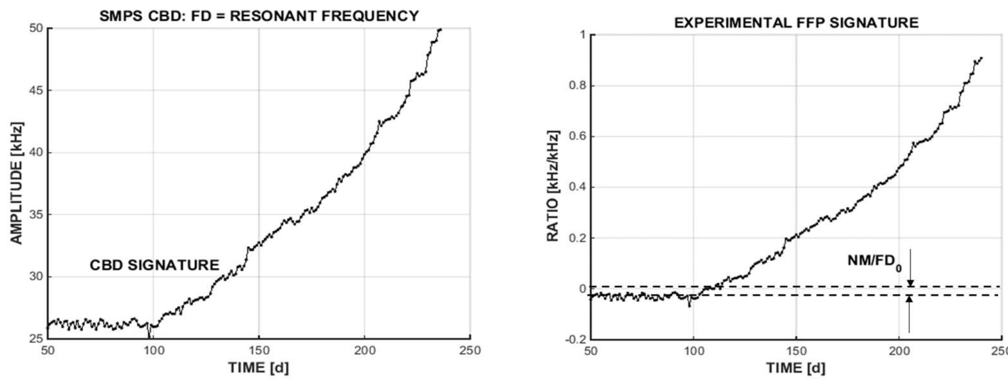


**Figure 2. FD extracted from CBD (left) and transformed into FFP data (right)**

***Degradation Progression Signature (DPS)***

A DPS is a collection of transformed FFP signature points: it is a function of a change in value, $dP$, of a parameter of interest $P_0$: (1) when there is no degradation, the value of the parameter is unchanged and $dP = 0$; (2) as degradation progresses, the magnitude of $dP$ increases:

$$\boldsymbol{DPS} = \{DPS_{m\,,}\, DPS_{m+1}, \ldots, DPS_{m+n}\} \tag{9}$$

$$DPS_i = dP_i / P_0 \tag{10}$$

A major advantage of DPS data is that its characteristic curve is defined by $dP_i/P_0$ and when that is linear, which is often the case, the DPS is linear. Even when $dP_i/P_0$ is not linear, a DPS signature curve is more linear than its underlying FFP signature.

## 2. Transforming FFP Signatures into DPS

Degradation-related signatures result from failure modes that can be modeled as power functions or exponential functions (Table 1). To transform signatures from one form to another, the authors developed seven sets of degradation functions (Table 2): five are power functions and two are exponential functions, which have either increasing or decreasing amplitudes – constant signatures are only applicable in the sense that they indicate the absence of degradation. Signatures having decreasing amplitudes are transformed to a complementary, decreasing amplitudes are transformed to a complementary, increasing amplitude signature and degradation model. Degradation signatures are further classified by whether the signature has a decreasing or increasing slope angle as degradation progresses: a linear, straight-line degradation signature is a special case that is classified as having a constant slope angle. The slope angle is the arc between a tangent to the curve and the horizontal axis. Sample plots of degradation functions and their DPS transforms are shown in Figure 3 through Figure 9.

**Table 1. Degradation Functions**

| Function Set and Type | Decreasing Amplitude | Increasing Amplitude |
|---|---|---|
| Set 1: Power-function | $-(dP_i/P_0)^n$ | $(dP_i/P_0)^n$ |
| Set 2: Power-function | $1-[1/(1-dP_i/P_0)]^n$ | $[1/(1-dP_i/P_0)]^n-1$ |
| Set 3: Power-function | $[1/(1+dP_i/P_0)]^n-1$ | $1-[1/(1+dP_i/P_0)]^n$ |
| Set 4: Power-function | $1-(1+dP_i/P_0)^n$ | $(1+dP_i/P_0)^n-1$ |
| Set 5: Power-function | $(1-dP_i/P_0)^n-1$ | $1-\ (1-dP_i/P_0)^n$ |
| Set 6: Exponential-function | $1-\exp(dP_i/P_0)$ | $\exp(dP_i/P_0)-1$ |
| Set 7: Exponential-function | $\exp(-\,dP_i/P_0)-1$ | $1-\exp(-dP_i/P_0)$ |

**Table 2. Transform Models**

| Increasing FFP | Increasing DPS | DPS-based FFS |
|---|---|---|
| $\{\mathbf{FFP}\}=\{(\boldsymbol{FD_i}-\mathbf{FD_0})/\boldsymbol{FD_0})\}$ | $\{\mathbf{DPS}\}=\{(\boldsymbol{dP_i/P_0})\}$ | $\{\boldsymbol{FFS_{DPS}}\}=(\{\boldsymbol{DPS}\}/\boldsymbol{FL_{DPS}})\mathbf{100}$ |
| $\boldsymbol{FD_i}=\boldsymbol{FD_0}\ \boldsymbol{g}(\boldsymbol{dP_i},\boldsymbol{P_0})$ | $\boldsymbol{dP_i/P_0}\ =\boldsymbol{f}(\boldsymbol{FFP_i})$ | $\boldsymbol{FL_{DPS}}=\boldsymbol{h}(\boldsymbol{FL_{FFP}})$ |
| $\boldsymbol{g}(\boldsymbol{dP_i},\boldsymbol{P_0})$ | $\boldsymbol{f}(\boldsymbol{FFP_i})$ | $\boldsymbol{h}(\boldsymbol{FL_{FFP}})$ |
| $(dP_i/P_0)^n$ | $(FFP_i)^{1/n}$ | $(\ FL_{FFP})^{1/n}$ |
| $[1/(1-dP_i/P_0)]^n-1$ | $1-1/(FFP_i+1)^{1/n}$ | $1-1/(\ FL_{FFP}+1)^{1/n}$ |
| $1-[1/(1+dP_i/P_0)]^n$ | $1/(1-FFP_i)^{1/n}-1$ | $1/(1-FL_{FFP})^{1/n}-1$ |
| $(1+dP_i/P_0)^n-1$ | $(FFP_i+1)^{1/n}-1$ | $(FL_{FFP}+1)^{1/n}-1$ |
| $1-\ (1-dP_i/P_0)^n$ | $1-(1-FFP_i)^{1/n}$ | $1-(1-FL_{FFP})^{1/n}$ |
| $\exp(dP_i/P_0)-1$ | $\ln(FFP_i+1)$ | $\ln(FL_{FFP}+1)$ |
| $1-\exp(-dP_i/P_0)$ | $\ln(1/(1-FFP_i))$ | $\ln(1/(1-FL_{FFP}))$ |

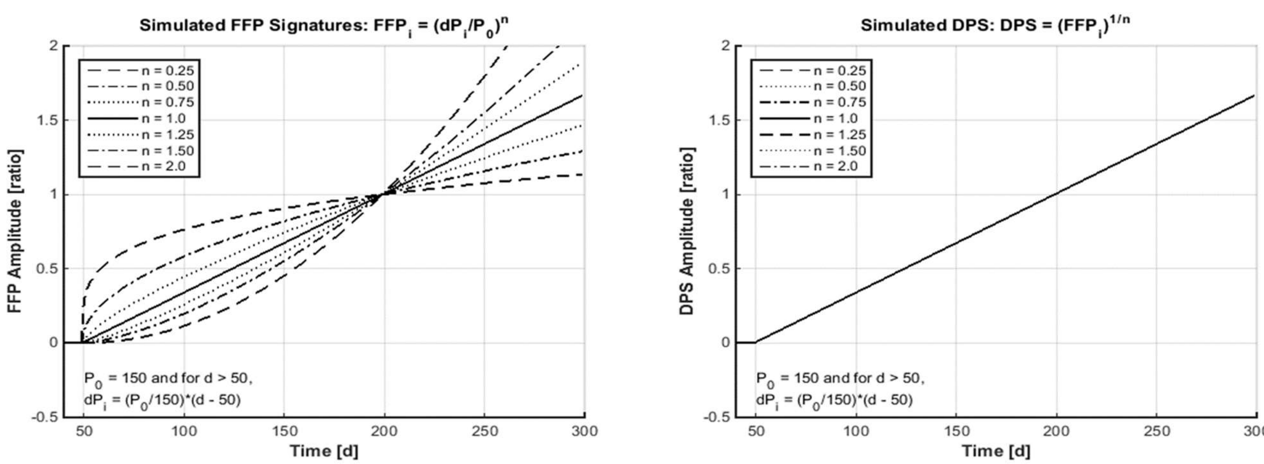


**Figure 3. Example FFP and DPS plots for $\boldsymbol{f(dP_i,P_0)=(dP_i/P_0)^n}$**

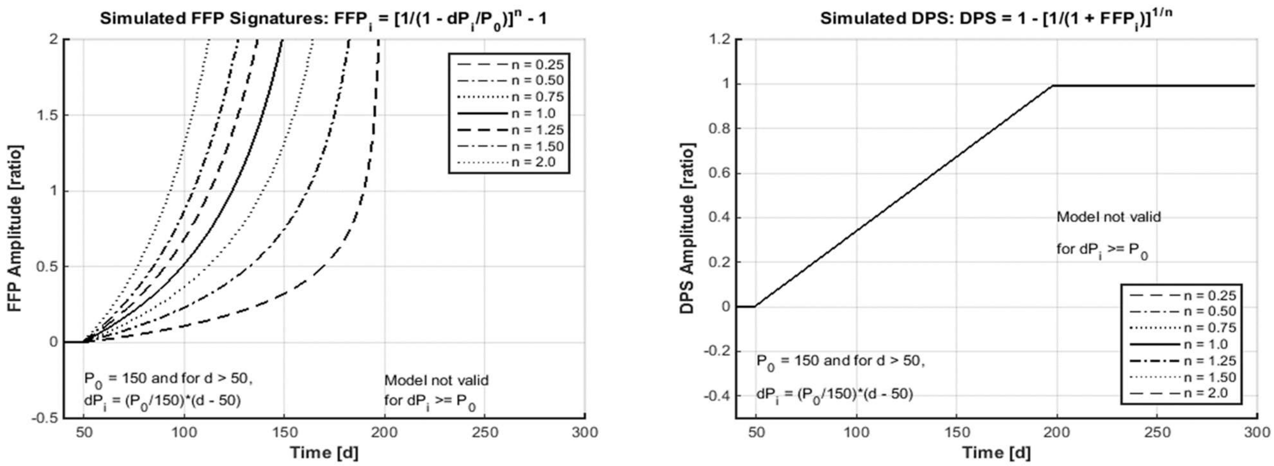


**Figure 4. Example FFP and DPS plots for $f(dP_i, P_0) = [1/(1 - dP_i/P_0)]^n - 1$**

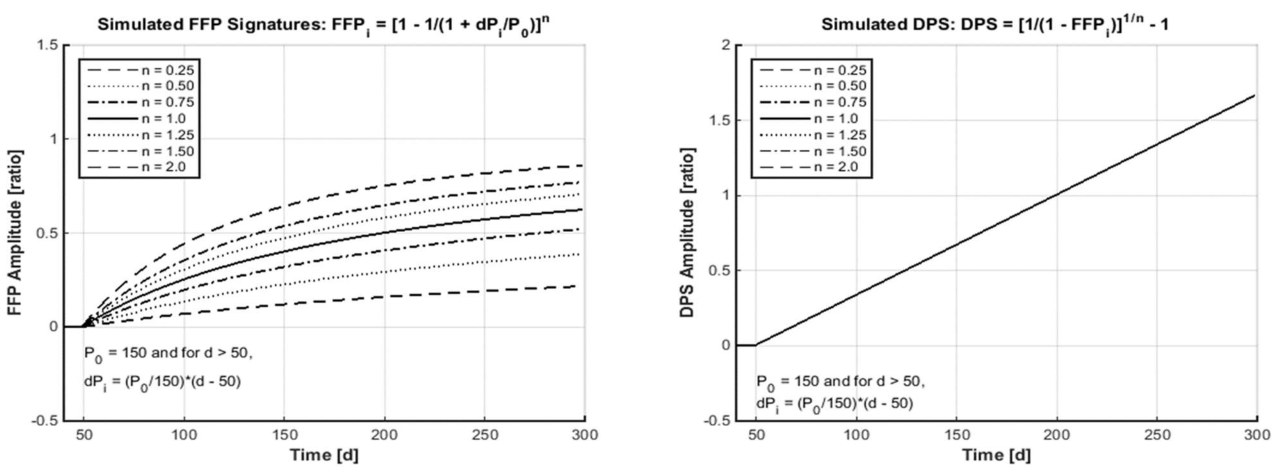


**Figure 5. Example FFP and DPS plots for $f(dP_i, P_0) = 1 - [1/(1 + dP_i/P_0)]^n$**

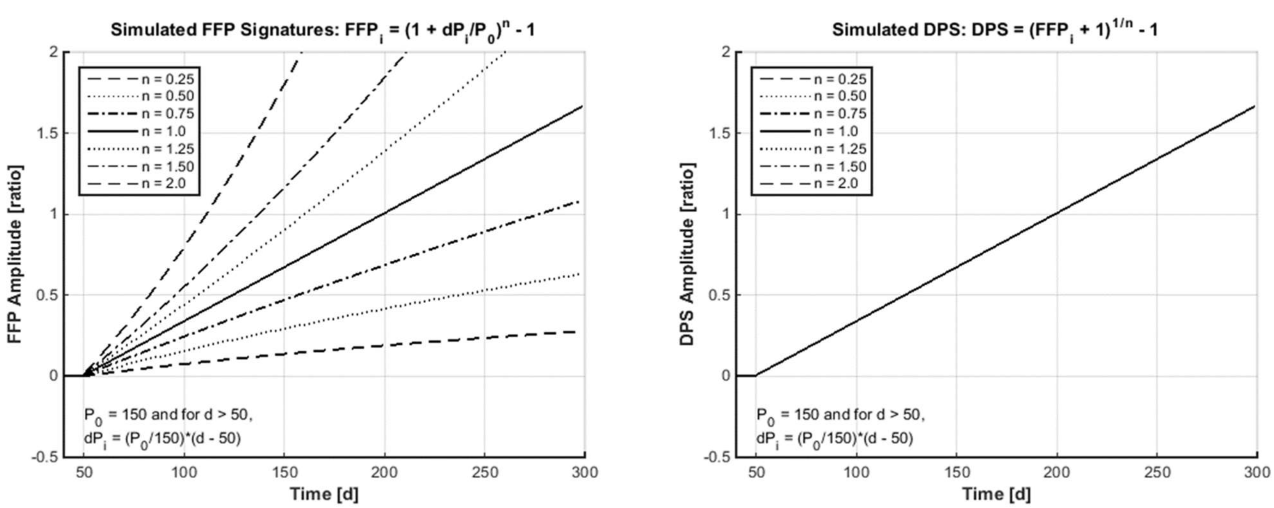


**Figure 6. Example FFP and DPS plots for $f(dP_i, P_0) = (1 + dP_i/P_0)^n - 1$**

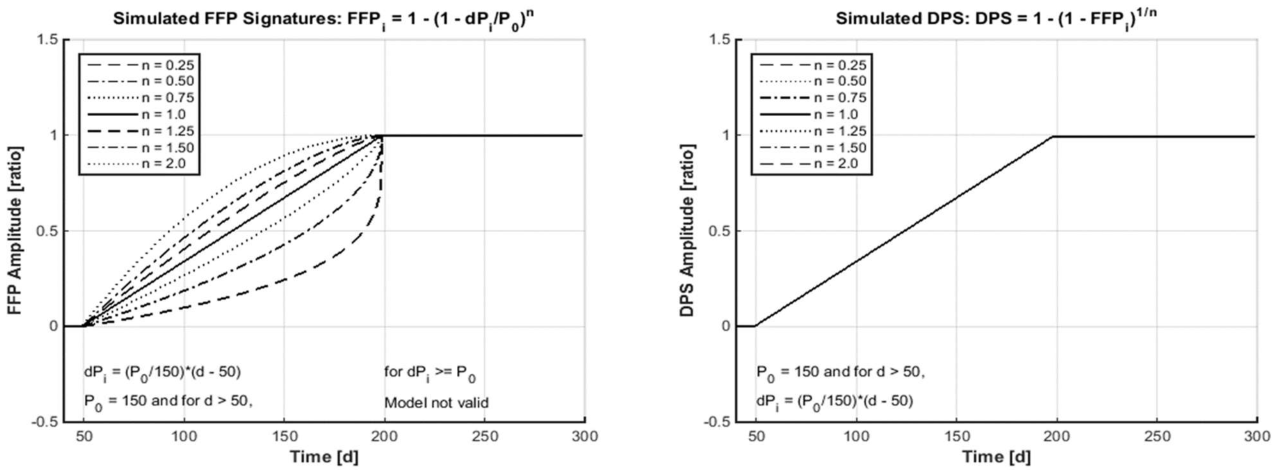


**Figure 7. Example FFP and DPS plots for $f(dP_i, P_0) = 1 - (1 - dP_i/P_0)^n$**

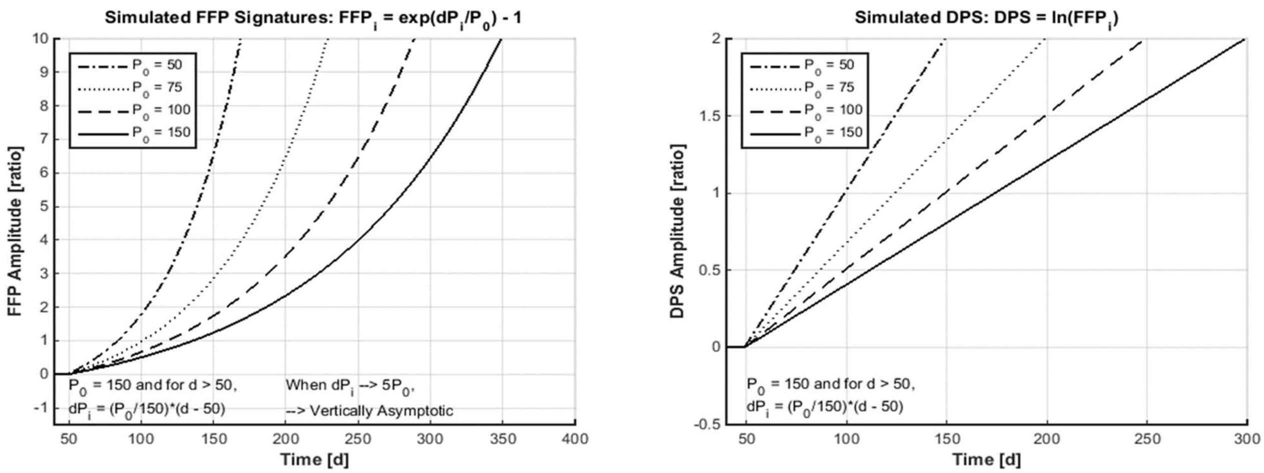


**Figure 8. Example FFP and DPS plots for $f(dP_i, P_0) = \exp(dP_i/P_0) - 1$**

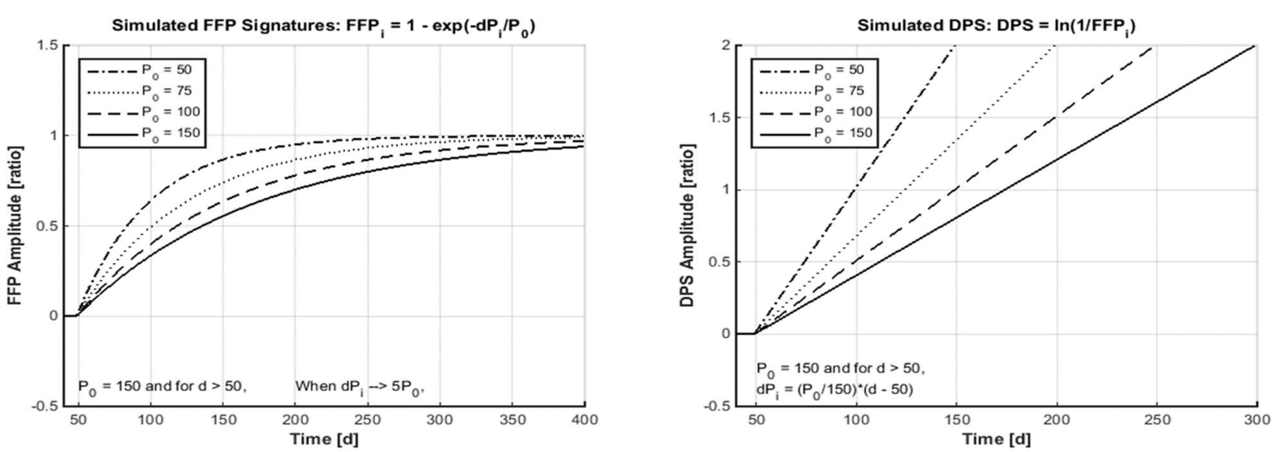


**Figure 9. FFP and DPS plots for $f(dP_i, P_0) = \exp(dP_i/P_0) - 1$ and $f(dP_i, P_0) = 1 - \exp(-dP_i/P_0)$**

## 3. TRANSFORMING DPS SIGNATURES INTO FUNCTIONAL FAILURE SIGNATURES (FFS)

The term 'functional failure' rather than simply 'failure' or 'physical failure' should be used in prognostics. Functional failure means reaching a level of degradation at which a device or component and the assembly in which it is located no longer operates within specifications: functional failure is not absolute: it is dependent not only on the device, component, and assembly, but also on the application and the requirements for that application. A functional-failure signature (FFS) is obtained by doing the following:

1. For each failure mode of interest, collect historical data and/or experimental data and/or simulation data.
2. Transform that data (CBD) into FFP signature data using Eq. (5):

   $$FFP_i = (FD_i - FD_0 - NM)/FD_0 \quad .$$

   FFP signatures reduce modeling complexity because of the follow ing: (1) common amplitude units-of-measure – a relative ratio, (2) ratio values of zero or less are indicative of no degradation – use a sufficiently large value of NM, and (3) values greater than zero are indicative of degradation – a value greater than 1.0 indicates the signal has more than doubled in magnitude.
3. Plot the FFP signature then compare and choose one, or maybe two, signature models from those shown in Figure 3 through Figure 9. For example, consider the example shown in Figure 10:

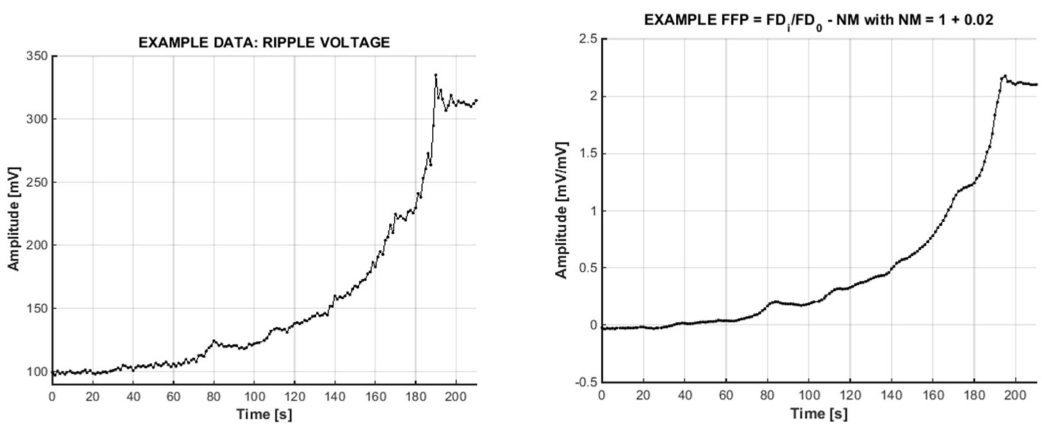


**Figure 10: Example CBD-based and FFP signatures**

Instead of using physics of failure (PoF) and failure-mode effects analyses (FMEA) to determine the effects of the failure on CBD and then solving for $dP_i/P_0$, use a heuristic approach: assume the solution is one of those listed in Table 2. For the example shown in Figure 10, by inspection and matching of curves, there are two candidate solutions:

$$DPS_i = 1 - 1/(FFP_i + 1)^{1/n} \tag{11}$$

and

$$DPS_i = \ln(FFP_i + 1) \tag{12}$$

Which results in the plots shown in Figure 11: the left-hand plots using n = 0.5 or n =

1.0 are more linear than the right-hand plot.

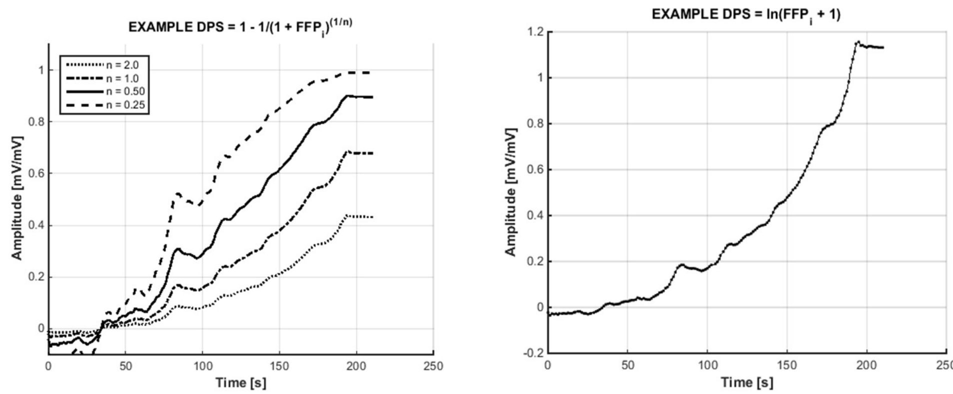


**Figure 11. Plots from using Eq. 11 (left side) and Eq. 12 (right side)**

4. Select or otherwise determine a value of FFP at which the prognostic-enabled device, component, or assembly is defined as functionally failed: $FL_{FFP}$ = FFP value at failure. That defines the level of degradation at which the device or component is no longer capable of operating within specifications. Then calculate the equivalent failure DPS threshold. For example, suppose you decide to use $FL_{FFP}$ = 0.90 and n = 0.5 or 1.0, then from Table 2:

$$FL_{DPS} = h(FL_{FFP}) = 1 - 1/(\,FL_{FFP} + 1)^{1/n} \tag{13}$$

$$FL_{DPS} = 0.72 \text{ for n} = 0.5 \text{ and } FL_{DPS} = 0.47 \text{ for n} = 1.0 \quad .$$

5. Then transform DPS data into FFS data, which results in the plots shown in Figure 12: note the two plots intersect each other at FFS amplitudes of 0 and 100%.

$$\boldsymbol{FFS}_{\text{DPS}} = (\boldsymbol{DPS}/FL_{DPS})100 \tag{14}$$

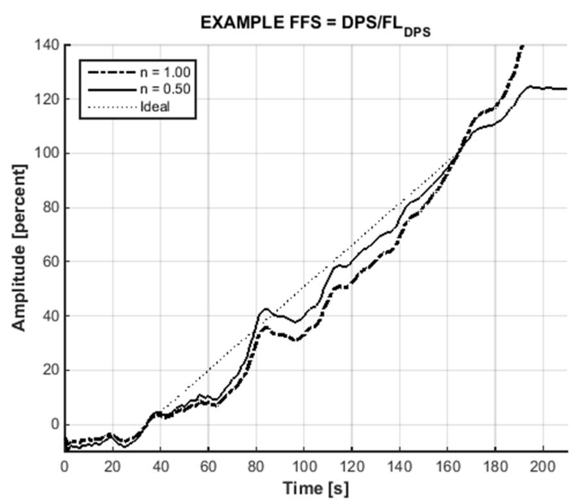


**Figure 12. Example FFS plots**

Evaluate the **FFS** plots and determine whether to slightly change the value of n: for n = 0.40

instead of 0.50, the **FFS** plot is shown in **Error! Reference source not found.**. Compare that to an ideal FFS transfer curve – the dotted straight line.

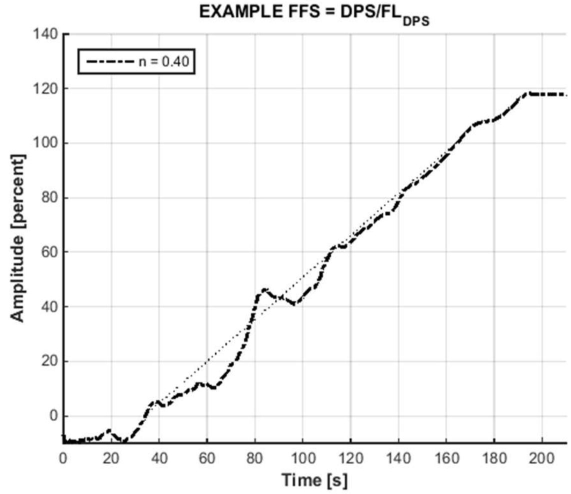


**Figure 13. Final FFS plot using Eq. 14 with n = 0.40 in Eq. 11 and Eq. 14**

### 4. VERIFY RESULTS

We define FFS nonlinearity (FNL) as a measure of the deviation between an actual FFS and an ideal FFS: very much like an integral nonlinearity (INL) assessment of an analog-to-digital data converter or transducer output transfer curve. A point-by-point FNL is a measurement of the error between an actual FFS value and the ideal FSS value at that point in time and is obtained using the following [6]-[9]:

$$TTF = t_{FAILURE} - t_{ONSET} \qquad \text{calculate time-to-failure} \tag{15}$$

then for each point in the set $\{FFS_i\}$, we create an ideal point,

$$IDEAL\_FFS_i = 100\,(t_i - t_{ONSET})/TTF \tag{16}$$

and calculate point-by-point nonlinearity and total nonlinearity:

$$FNL_i = FFS_i - IDEAL\_FFS_i \tag{17}$$

Total FNL error ($FNL_E$) provides an assessment of the nonlinearity of the FFS curve:

$$FNL_E = \max(\text{abs}(\{FNL_i\})) \tag{18}$$

Figure 14 shows the non-linearity plot for transfer curve shown in Figure 13.

Despite the relatively large variation in the data between the approximate times of 60 and 90 seconds, the $FNL_E$ is less than 13 percent, which in the assessment of the authors is quite remarkable. This data, when input to a prediction algorithm employing estimation techniques such as Kalman filtering, is very likely to produce very accurate prognostic information.

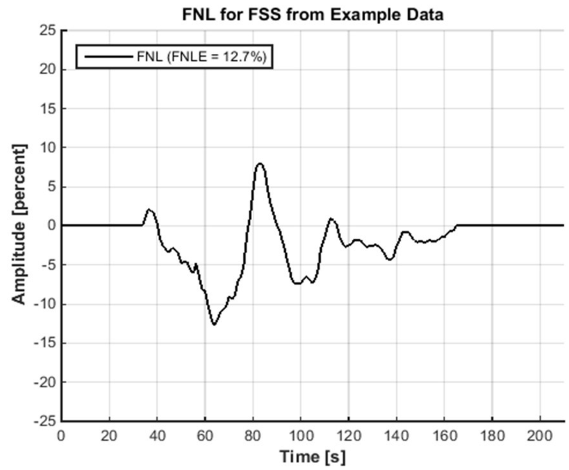


**Figure 14. Non-linearity for FFS shown in Figure 13**

## 5. Summary

In this paper a concept of signatures is introduced and an example given to illustrate CBD features that can be extracted as FD to create an FFP signature, which can be further transformed into DPS data and then into FFS data – one data point at time. Instead of rigorous, time-consuming, and error-prone PoF and FMEA analysis to derive transformation models, the paper describes a heuristic-based approach for advantageous used of a set of already derived models (Table 1 and Table 2) to transform CBD into FFS data for use as a transfer curve for producing prognostic information. An FFS progresses from 0 percent or less (no degradation) to 100 percent or higher (functionally failed). Next a method is introduced to evaluate non-linearity of FSS data (FNL): that method is based one used for evaluating the nonlinearity of data converters.

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