

# Improved Reliability-based Decision Support Methodology Applicable in System-level Failure Diagnosis and Prognosis

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**Abstract**—Reliability modeling and troubleshooting reasoning involving complex component interactions in complex systems are an active research topic and a critical challenge to be overcome in decision support. In this paper, we propose an innovative concept of decision support methodology for system failure diagnosis and prognosis in complex systems. Advanced causal structure incorporating domain and engineering knowledge and a new Bayesian network (BN) representation of system structure and component interaction are proposed. Based on the BN representation, a Bayesian framework is developed to analyze and fuse the multi-source information from different hierarchical levels of a system. This capability supports higher fidelity modeling and assessing the reliability of the components, subsystems and the system as a whole. The feasibility of our advanced causal structure approach has been proven with implementation using test data acquired from electromechanical actuator (EMA) systems. A case study is successfully conducted to demonstrate the effectiveness of the proposed methodology. The proposed decision support process in integrated system health management (ISHM) will enable enhancements in flight safety and condition-based maintenance (CBM) by increasing availability and mission effectiveness while reducing maintenance costs.

**Index Terms**— Failure and fault detection/isolation, Reliability analysis, Bayesian network, Decision support, Information fusion, Causal analysis, Troubleshooting

## I. INTRODUCTION

**R**IGOROUS routine inspection and maintenance for today's complex systems (e.g., fixed and rotary wing aircraft) are performed to ensure the health of the plane's numerous mechanical and electronic systems. While vital, this constant process has seen significant cost increases over the past 10 years as various cost components such as labor, parts, and aircraft downtime rise in conjunction with the increasing complexity and aging of these systems. These trends have compelled the development of an effective troubleshooting and

decision support system for complex system faults/failures. It ultimately provides substantial benefits to the maintainer of complex systems and maintenance facilities.

Complex engineering systems are characterized by a multilevel hierarchical physical structure that embraces a large number of components interconnecting and interacting with each other, jointly contributing to the functionality of a subsystem, and a large number of such subsystems interconnecting and interacting with each other jointly contributing to the functionality of the system. This characterizes a hierarchical system with three levels, i.e., the component, subsystem, and system levels, whereas the general hierarchical system under investigation in this paper can include any number of levels.

As multilevel hierarchical systems, including but not limited to, a ship hull assembly [3], a bridge [4], and an anti-aircraft missile system [5] are usually deployed in environments associated with enormous financial investment, or high-level national security, failures of such systems in use are devastating. This makes system-level diagnostic and prognostic decision support of multilevel hierarchical systems extremely important. The challenges, on the other hand, are also enormous: First, there are usually very few or no data available at the higher-level subsystems or systems, since pre-launch whole-system reliability testing may be infeasible or too expensive. Second, although data and prior knowledge may be available for the subsystems and components, how to analyze and fuse all the information to predict the stability and reliability of the system remains a challenging problem. Especially due to globalization, production of different components may be carried out at different industrial sites and used for assemblies of various subsystems. As a result, it is possible that the data and knowledge about the reliability of the subsystems and the components contain conflicting information. For example, the reliability of a subsystem can be assessed using its test data. It can also be predicted based on the test data of the components of this subsystem and the physical interconnecting relationship of the components (e.g., serial or parallel configuration). As subsystem assembly and component production may be carried out in different locations, it is very likely that the reliability assessments/predictions by the two methods mentioned above will give different results. Third, due to the possibly large number of components a subsystem consists of and their complex interacting relationships,

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especially considering that the components work under the same environment when the system is in use, failures of the components are usually interdependent. For example, Langseth and Portinale [30] studied the reliability of a gas turbine controller and found that when the “power supply” works abnormally, it induces anomalous behaviors in the components “supply equipment of the main controller” and “back-up unit.” This is because the malfunction of “power supply” causes other components to work more intensively, thus expediting aging, degradation, and eventual failures. Failure interdependency among components and subsystems adds to the complexity of the reliability modeling of the system. Based on these reasons and to tackle the aforementioned challenges, in this paper, we propose an innovative concept for a new decision support methodology for system failure diagnosis and prognosis in complex systems.

#### A. Related Works

As summarized in [37], existing failure diagnosis and prognosis methods include case-based methods [38], rule-based methods [39], artificial intelligence-based methods [16] and model-based methods [13], [36]. As compared to other methods, model-based methods represent system structure with graph-based models such as Bayesian network to enable the precise quantification of failure interaction relationship among system elements. The proposed work essentially uses a model-based method and, compared to existing work, it intersects with three areas in the literature.

The first area is constructing advanced causal structure by incorporating domain and engineering knowledge. For each component/subsystem, the knowledge should cover 1) working physics and working conditions, 2) health status indicators, 3) functionalities and performance indicators, 4) potential failure modes and effects, 5) corresponding causes of the failure modes, and 6) corresponding mitigation methods of the failure modes. There is existing work on constructing causal structure with Bayesian networks (BNs) [7], [8], [9], [15]. However, the existing work does not consider domain and engineering knowledge to provide capability and flexibility in handling a complex system that has too little reliability test data.

The second area is reliability modeling of multilevel systems. Existing work along this line has studied systems with various connection structures with binomial test data [10] and exponential lifetime data [11]. However, the existing modeling methodologies assume that the failures of components/subsystems are independent.

The third area is BNs in reliability modeling. BNs provide an effective model for characterizing the dependent and independent relationships among the variables in a domain. In reliability modeling, BNs have been extensively used to model the interacting relationships among the components of a system [7]. Other related domains that utilize BNs include software reliability [12], fault-finding systems [6], [35], and maintenance modeling [29]. However, the existing work noted above does not consider systems with multilevel hierarchical structures. Recently, BN formalism from FT [14] and BN hierarchical fault diagnosis model [36] applicable in the

presence of a large number of components and subsystems were presented. They provide hierarchical decomposition framework with component dependencies and demonstrate the effectiveness by using synthetic data in their application. However, the method available in their paper is not applicable to our problem. One of the main differences in developing advanced causal structure is that we incorporate domain and engineering knowledge.

The work for constructing advanced causal structure is based on our preliminary work [1] with a constraint-based learning method to build causal analysis with BNs. We extend this work and offer advanced causal structure. In addition, we propose a method for reliability modeling of multilevel hierarchical systems with interdependent subsystems/components by fusing the data and prior knowledge collected at all levels of the system. Specifically, we propose the use of a BN to model the failure interdependency in the system. Furthermore, we develop a Bayesian framework to analyze and fuse the multi-source information regarding the reliability of a subsystem (or the system), including the test data and prior knowledge about this subsystem as well as the information propagating from the lower level.

The remainder of this paper is organized as follows: Section II explains the requirement of decision support reasoning with interconnected submodules. In section III, we describe the development of multilevel methodology for reliability modeling and troubleshooting reasoning of hierarchical systems. Section IV explains multilevel information analysis and fusion development. Implementation and the case study to demonstrate the effectiveness of the proposed methods are presented in section V. Integrated system health management (ISHM) regarding diagnosis support is discussed in section VI. Section VII concludes the paper.

## II. REQUIREMENT OF DECISION SUPPORT REASONING

Our reliability modeling and troubleshooting reasoning integrates the engineering and domain knowledge of different levels of an engineering system with the statistical analysis results of the data collected to identify and/or predict the failures of subsystems and components, and evaluate their potential effects. The reasoning is basically divided into the following interconnected submodules.

#### A. Anomaly Detection

This submodule detects the failure of components, subsystems or even systems, and indicates that “something failed” in the system. Statistical process control concepts and charting techniques are employed in this submodule to monitor the system performance based on the performance measurement data collection from the component, subsystem, and system level. One byproduct of the anomaly detection model is the normal performance measurement of a statistically stable component, subsystem, and even system. Our data-driven approach for anomaly detection can be found in detail in [2].

### B. Failure Diagnosis and Isolation

This submodule answers the questions of “what failed” and “why it failed.” It locates the component(s) and subsystem(s) that caused the system failure and identifies the root cause of the failure by mapping the statistical patterns extracted from multivariate data with the engineering knowledge representation of failure physics. A mathematical model, such as a factor analysis model and/or Bayesian network model, is built to link the root causes of failure with the performance measurement.

### C. Failure Prognosis

This submodule answers the question “What will fail?” Based on the normal performance measures identified in anomaly detection and the mathematical cause-effect model learned in failure diagnosis, the prediction of potential failure, given current data and current status of the component/subsystem, can be implemented. Most importantly, the cause(s) of the predicted failure are also identified.

In this paper, we focus on implementation of the failure diagnosis and prognosis requirement.

## III. HIERARCHICAL SYSTEM ANALYSIS METHODOLOGY

This paper proposes multilevel methodology for reliability modeling and troubleshooting reasoning of hierarchical systems. Our advanced causal structure and a new BN are developed to represent the system structures and a Bayesian framework is employed to aggregate the lower-level information to upper-level modeling.

### A. Development of Flow Diagram with Domain Knowledge

The fault tree (FT) is a commonly used method for quantitative risk modeling. It is one of popular methodologies for evaluating failure occurrences in safety-critical systems [31] in a top-down fashion. We will use the former term, because we focus on decision support issues in complex system troubleshooting. The system failure is often represented by the event at the top. It is decomposed into basic events that describe detailed causes and basic components’ failures. Logic gates like AND or OR provide the logic expressions among different failure events. Given the probabilities of basic events and the logic structure of the tree, the probability of system failure can then be calculated.

The FT structure represents existing domain knowledge of failure event relationships in the system. In order to construct the fault tree, the rules-based knowledge (e.g., Failure Modes and Effects Analysis (FMEA)) should be transformed into numerical entities in facilitating graphical representation of the FT. The rules-based domain knowledge is first transformed into the adjacency matrix.

Fig. 1 shows an example of partial implementation of the FT in a graphical representation. The top event refers to the failure of the EMA overall system. OR logic gates are used to represent failure events’ relationships based on domain knowledge. The efforts of FT development allow us to provide advance causal structure for troubleshooting and decision support with causal analysis, and will be explained more in the

implementation process.

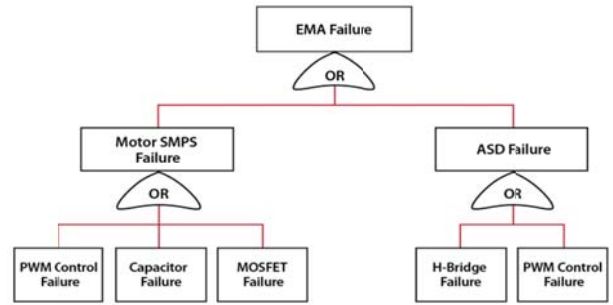


Fig. 1. An example of FT graphical representation

### B. Clarification of Relationships Between Flow Diagram and Bayesian Network

Unlike the tree-based structure of FT, BNs [17] are Directed Acyclic Graphs (DAG) for representing the joint distribution and reasoning under uncertainty. BNs have been applied in many areas for probabilistic reasoning [18], [19], [20], decision making [21], [22], robust localization [32], and constructing causal structure [7], [8], [9].

Reasoning under uncertainty refers to the capability of representing and drawing an inference of data with uncertainty. In practice, collected data may include noise, errors, or even missing values. Moreover, knowledge provided by experts may sometimes contain ambiguity or non-informative guesswork. It is noted that uncertainty in this paper refers to the randomness of the reliability and failure relationship among system elements. The reliability of a system element is treated as a random variable to quantify its subjective belief. The failure relationship is represented with conditional probabilities to provide a more generic and realistic representation. Uncertainty may influence both the accuracy and precision of reliability estimation. High accuracy of the reliability estimation means that the expected reliability has small or no deviation from its true value. High precision means the variance of the estimated reliability is small. Although in the FT every basic failure event is associated with a probability, the logic gates are deterministic and there is no belief-updating process as in the Bayesian inference scheme adopted by BNs. In BNs, each node is assigned by a random variable with certain probability distribution in describing a particular event. Probability distribution can fully capture the uncertain information of the data. Arcs direct from parent nodes to children nodes in indicating the cause-effect relationships. Conditional probability tables (CPTs) are specified for each pair of connected nodes in quantifying the dependency strength. CPT is rather flexible in handling logic relationships with uncertainty.

Incorporating domain and engineering knowledge into our new BN, we provide a more powerful and flexible representation of dependency among different variables by solving some of the limitations (e.g., undirected arcs and connection between nodes) that we found in [1].

### C. A BN Representation of Hierarchical System Structure

A BN is used to model the interdependency in the system. A

general BN includes a structure called a DAG and a set of parameters. The DAG consists of nodes which are random variables and directed arcs. An arc pointing from node A to node B indicates that A is a direct cause (parent) of B, where “direct” means that the causal influence from A to B is not mediated by other nodes in the BN. A directed path (a directed arc or a set of singly connected arcs with the same direction) from A to B indicates that A is a cause (called an ancestor) of B. According to this definition, an ancestor can be a parent or an indirect cause. Furthermore, the parameters of a BN include the conditional probability distribution of each node given its parents. When the nodes are categorical variables, the parameters can be specified by the conditional probability mass function (PMF) of each node.

To adapt the general BN to our problem, we propose the following new definitions and notations. In this paper, a component or a subsystem of a hierarchical system is referred as an “*element*.” When a BN is used for a hierarchical system, each node represents an element. Specifically, let node  $X_{l,j}$  be the health status of the  $j^{\text{th}}$  element at the  $l^{\text{th}}$  level, where  $l=1, \dots, L$ ,  $j=1, \dots, N_l$ , and  $L$  and  $N_l$  are the total number of levels of the system and the total number of elements at level  $l$ , respectively. To make BNs an appropriate representation for hierarchical systems, the following restrictions are further defined on the directed arcs:

- (i) Arcs can only point from lower levels to higher levels, but not reverse. The reason for this restriction is based on the assumption that failure of higher-level element (e.g., a subsystem) is caused by its composing lower-level element (e.g., one of its components).
- (ii) There must be a directed path from each component/subsystem to the system, i.e., we exclude nodes that do not exert any causal influence (directly or indirectly) on the system from the BN.

A portion of a generic BN representation of multilevel hierarchical systems is given in Fig. 2. Furthermore, this paper focuses on binomial failures for an element, i.e., each node,  $X_{l,j}$ , in the BN can take values 0 or 1 to represent the status “working” or “failure,” respectively. Then, for a node with  $K$  parents, the parameters, i.e., the conditional PMF, includes  $2^K$  probabilities. For example, consider node  $X_{l,1}$  in Fig. 2. Because  $X_{l,1}$  has two parents  $\{X_{l-1,1}, X_{l-1,2}\}$ , its parameters include  $2^2$  probabilities, including  $P(X_{l,1}=1 | X_{l-1,1}=0, X_{l-1,2}=0)$ ,  $P(X_{l,1}=1 | X_{l-1,1}=1, X_{l-1,2}=0)$ ,  $P(X_{l,1}=1 | X_{l-1,1}=0, X_{l-1,2}=1)$ , and  $P(X_{l,1}=1 | X_{l-1,1}=1, X_{l-1,2}=1)$ .

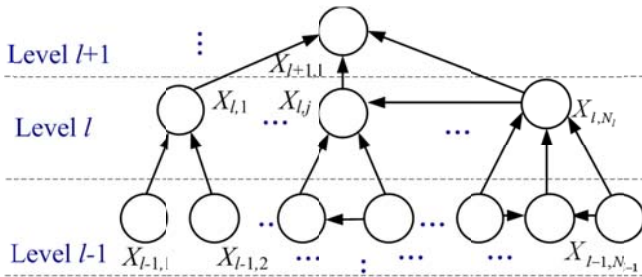


Fig. 2. A BN representation of a hierarchical system

Note that the popular serial and parallel systems are just special, deterministic cases of the BN representation. For example, if subsystem  $X_{l,1}$  is a parallel system, it simply means that  $P(X_{l,1}=1 | X_{l-1,1}=0, X_{l-1,2}=0) = 0$ ,  $P(X_{l,1}=1 | X_{l-1,1}=1, X_{l-1,2}=0) = 0$ ,  $P(X_{l,1}=1 | X_{l-1,1}=0, X_{l-1,2}=1) = 0$ , and  $P(X_{l,1}=1 | X_{l-1,1}=1, X_{l-1,2}=1) = 1$ . If subsystem  $X_{l,1}$  is a serial system, it simply means that  $P(X_{l,1}=1 | X_{l-1,1}=0, X_{l-1,2}=0) = 0$ ,  $P(X_{l,1}=1 | X_{l-1,1}=1, X_{l-1,2}=0) = 1$ ,  $P(X_{l,1}=1 | X_{l-1,1}=0, X_{l-1,2}=1) = 1$ , and  $P(X_{l,1}=1 | X_{l-1,1}=1, X_{l-1,2}=1) = 1$ . Here, it is very important to note that the BN representation is different from the reliability diagram, e.g., although  $X_{l-1,1}$  and  $X_{l-1,2}$  are drawn parallel to each other, they may compose a serial system. Furthermore, BN provides a more flexible mechanism of representing the probabilistic interrelationships among elements. The failure interdependency between elements on the same level can be incorporated in the conditional probability.

#### IV. MULTI-LEVEL INFORMATION ANALYSIS AND FUSION FRAMEWORK

For each node in the BN presented in Fig. 2, there will be three types of information sources that need to be fused, including the test data in terms of failure or survival, prior knowledge about this node in terms of Beta prior distribution parameters, and the information propagated from its ancestors (i.e., the lower-level elements) in terms of additional inputs to the prior distribution parameters. Here, the fusion considers only the information at the “immediate” lower level but not all lower levels due to Restriction (ii) in Section III.C.

Because information fusion at each node follows the same manner, we will focus on a subsystem,  $X_{l,j}$ , to present the proposed method. The method can easily be applied to the elements at any levels. Note that since components are at the lowest level of a hierarchical system, there is no propagated information considered. Therefore, only two sources of information, i.e., the test data and prior knowledge, will be fused. This is just a special case of the proposed method.

##### A. Information Integration of Single Element

Consider a binomial distribution model for the test data regarding  $X_{l,j}$ . Such data can be obtained from success-failure reliability tests, where a sample of test units is tested within certain pre-determined time duration and the number of survivors and failures after the test is recorded. Such data can evaluate the average reliability within the time duration, and therefore, the average reliability quantity that will not involve time. Given that  $n$  independent tests are conducted and the reliability (i.e., the survival probability) of each test is  $r$ , the number of survivors,  $s$ , follows a binomial distribution, i.e.,  $s \sim \text{binomial}(r, n)$ . The prior knowledge about  $X_{l,j}$ , is in the form of a prior distribution for  $r$ . A beta prior distribution is commonly adopted. The rationales are: (i) a beta prior is the conjugate prior for binomial data, and thus, results in computational advantages; (ii) the beta family is rich in shape, thus providing a flexible representation for various types of prior knowledge; (iii) the beta family is the best choice for use in determining the most conservative prior when there is little prior knowledge available [24]; and (iv) the effect of assuming

a beta prior in binomial data, when the true prior is not beta, is negligible in many practical applications [23]. Let  $r$  follow a beta prior distribution, i.e.,  $r \sim \text{beta}(s^0+1, n^0-s^0+1)$ . The parameters of this distribution are denoted by  $s^0+1$  and  $n^0-s^0+1$  for the ease of interpretation, which is that, when  $s^0$  and  $n^0$  are integers,  $s^0+1$  and  $n^0-s^0+1$  are the numbers of prior survivors and failures, respectively. And their sum,  $n^0+2$ , is the total number of prior tests [25]. With this conjugate prior distribution, the posterior distribution of  $r$  is also beta, i.e.,

$$r \sim \text{beta}(s + s^0 + 1, n + n^0 - s - s^0 + 1). \quad (1)$$

### B. Induced Prior Propagation

Note that what has been described above is how to apply standard Bayesian inference for fusing the test data and prior knowledge regarding  $X_{l,j}$ . Full information fusion needs to incorporate the information propagated from the components of  $X_{l,j}$ . In this paper, we consider the propagated effects as another ‘‘prior’’ knowledge for  $X_{l,j}$ , achieved from investigating the reliability of its composing subsystems or components. To distinguish, this prior is called the ‘‘induced prior’’ and the prior discussed previously is called the ‘‘native prior.’’ We will discuss how to transform the propagated reliability information into the induced prior for  $X_{l,j}$ . Considering that the element  $X_{l,j}$  consists of  $p_{l,j}$  lower-level elements,  $\{X_{l-1,1}, \dots, X_{l-1,p_{l,j}}\}$ . Let  $\mathbf{PA}(X_{l,j})$  denote the parents of the element  $X_{l,j}$ , i.e.,  $\mathbf{PA}(X_{l,j}) \subseteq \{X_{l-1,1}, \dots, X_{l-1,p_{l,j}}\}$ . According to the Law of Total Probability, the reliability of  $X_{l,j}$  is

$$P(X_{l,j}=0) = \sum_h \alpha_{(l,j),h} P(\mathbf{PA}(X_{l,j})=h) \quad (2)$$

where where  $\alpha_{(l,j),h} \equiv P(X_{l,j}=0 | \mathbf{PA}(X_{l,j})=h)$ , and  $h$  is used to index each combination of values for the parents  $X_{l,j}$ . For example, consider the subsystem  $X_{l,1}$  in Fig. 2, which has two parents  $\{X_{l-1,1}, X_{l-1,2}\}$ , and  $h = \{\{0,0\}, \{0,1\}, \{1,0\}, \{1,1\}\}$ . In this paper, we assume  $\alpha_{(l,j),h}$  to be constants known from design or statistical analysis. For example, in special cases of a serial or parallel system,  $\alpha_{(l,j),h}$  can be 0 or 1. Furthermore, according to the Decomposition Theory of BN,  $P(\mathbf{PA}(X_{l,j}))$  can be decomposed into a product of the probability of each variable in  $\mathbf{PA}(X_{l,j})$  given its parents. Specifically, let  $PA_i(X_{l,j})$  denote the  $i^{\text{th}}$  parent of  $X_{l,j}$ ,  $i=1, \dots, p_{l,j}$ , i.e.,  $\mathbf{PA}_i(X_{l,j}) \in \mathbf{PA}(X_{l,j})$ . Then,

$$P(\mathbf{PA}(X_{l,j})=h) = \prod_i P(PA_i(X_{l,j})=k_i | \mathbf{PA}(PA_i(X_{l,j}))=\mathbf{k}_i), \quad (3)$$

where  $\kappa$  is the value of  $PA_i(X_{l,j})$  in  $h$ , and  $\mathbf{k}$  is a vector of the values of  $\mathbf{PA}(PA_i(X_{l,j}))$  in  $h$ . Eq. (3) indicates that  $P(\mathbf{PA}(X_{l,j})=h)$  is a function of the (conditional) reliability of each parent for  $X_{l,j}$ . For example, in Fig. 2,  $P(\mathbf{PA}(X_{l,1})=\{0,1\}) = P(X_{l-1,1}=0, X_{l-1,2}=1) = P(X_{l-1,1}=0) \{1 - P(X_{l-1,2}=0 | X_{l-1,1}=0)\}$ , where  $P(X_{l-1,1}=0)$  and  $P(X_{l-1,2}=0 | X_{l-1,1}=0)$  are the reliability and conditional reliability of  $X_{l-1,1}$  and  $X_{l-1,2}$ , respectively.

Therefore, as long as the (conditional) reliability of each parent for  $X_{l,j}$  is known, the reliability of  $X_{l,j}$  in (2) can be fully

specified. Considering the conditional reliability of the  $i^{\text{th}}$  parent of  $X_{l,j}$ ,  $P(PA_i(X_{l,j})=0 | \mathbf{PA}(PA_i(X_{l,j}))=\mathbf{k}_i)$ . If  $PA_i(X_{l,j})$  is a lowest-level element in a hierarchical system, only test data and prior knowledge are available and there is no propagated reliability information needing to be fused. The available two sources of information can be fused with standard Bayesian inference. Specifically, let  $s_{(i,l,j),\mathbf{k}}$  be the number of survivors out of  $n_{(i,l,j),\mathbf{k}}$  independent tests on  $PA_i(X_{l,j})$  while keeping its parents at the status specified by  $\mathbf{k}$ , i.e.,  $\mathbf{PA}(PA_i(X_{l,j}))=\mathbf{k}$ . We use a beta prior for the conditional reliability of  $PA_i(X_{l,j})$ , i.e.,

$$P(PA_i(X_{l,j})=0 | \mathbf{PA}(PA_i(X_{l,j}))=\mathbf{k}) \sim \text{beta}(s_{(i,l,j),\mathbf{k}}^0 + 1, n_{(i,l,j),\mathbf{k}}^0 - s_{(i,l,j),\mathbf{k}}^0 + 1), \quad (4)$$

where  $s_{(i,l,j),\mathbf{k}}^0 + 1$  and  $n_{(i,l,j),\mathbf{k}}^0 - s_{(i,l,j),\mathbf{k}}^0 + 1$  are integers and can be respectively interpreted as the numbers of prior survivors and failures in the hypothetical prior tests on  $PA_i(X_{l,j})$  when  $\mathbf{PA}(PA_i(X_{l,j}))=\mathbf{k}$ . Then, the posterior distribution for the conditional reliability of  $PA_i(X_{l,j})$  is

$$P(PA_i(X_{l,j})=0 | \mathbf{PA}(PA_i(X_{l,j}))=\mathbf{k}) \sim \text{beta}(s_{(i,l,j),\mathbf{k}} + s_{(i,l,j),\mathbf{k}}^0 + 1, n_{(i,l,j),\mathbf{k}} + n_{(i,l,j),\mathbf{k}}^0 - s_{(i,l,j),\mathbf{k}} - s_{(i,l,j),\mathbf{k}}^0 + 1). \quad (5)$$

Note that if a parent of  $X_{l,j}$  does not have any parent in the BN, its unconditional reliability can be specified in a similar way. After the posterior distribution for the (conditional) reliability of each parent of  $X_{l,j}$  is obtained, these posterior distributions can be plugged into (3), which is further plugged into (2) to compute the reliability of  $X_{l,j}$ . The reliability computed in this way integrates the information propagated from the lower-level elements and thus is the afore-defined induced prior for  $X_{l,j}$ .

Due to the complex functional form of the induced prior, it is very likely that this prior distribution is non-parametric. For computational simplicity, we adapt the approach by [26] to our problem by setting and approximating the exact induced prior with a beta distribution having the same first two moments. Use of the approximation is also for the convenience of combining the induced prior with the native beta prior distribution. Specifically, let the approximate beta distribution be  $P(X_{l,j}=0) \sim \text{beta}(b, c)$ , where the parameters  $b$  and  $c$  are derived as described in the Appendix.

### C. Native and Induced Priors Combination

Furthermore, the induced  $\text{beta}(b, c)$  and native  $\text{beta}(s^0+1, s^0-s^0+1)$  priors can be combined to generate a single beta prior for element  $X_{l,j}$ . In this paper, we follow the natural conjugates for generating the combined prior [23], i.e.,  $\text{beta}(w_1 b + w_2 (s^0+1), w_1 c + w_2 (n^0 - s^0 + 1))$ , where  $w_1$  and  $w_2$  are the weighting coefficients ( $w_1 + w_2 = 1$ ) assigned to the induced and native priors, respectively. For example, if the production of the components and the assembly of the components into the subsystem are carried out in two different sites, it may be appropriate to consider the induced and native prior information to be obtained independently. Dependency may

occur, for example, if the same group of domain experts is asked to assess the native prior of the subsystem as well as the priors of the components of this subsystem.

Finally, the combined prior distribution is integrated with the binomial test data on subsystem  $X_{i,j}$  to produce the posterior distribution for the reliability of  $X_{i,j}$ , i.e.,

$$\text{beta}(s+w_1b+w_2(s^0+1), n-s+w_1c+w_2(n^0-s^0+1)) \quad (6)$$

This completes the fusion of the three sources of reliability information of  $X_{i,j}$ . It is noted that  $X_{L,1}$ , denoting the system as a whole, is the only element at level  $L$ . This generic procedure can be applied iteratively to construct the multilevel reliability model for multilevel systems with any structure, taking advantage of the generic structure representation of BN and the generic Bayesian framework of the model parameter aggregation.

## V. IMPLEMENTATION PROCESS AND CASE STUDY

In this section we provide implementation of advanced causal structure by incorporating domain and engineering knowledge, and we demonstrate the proposed procedure of the reliability modeling of hierarchical systems. While this paper mainly focuses on the new implementation of advanced causal structure, causal analysis with BNs and various test cases can be found in detail in [1]. In that article we included and evaluated the effectiveness and accuracy of constructed causal Bayesian network (CBN) representation by comparing the number of causal relationships from trained data and original structures, and showed that our constructed CBN is highly effective and accurate in the targeted system.

### A. Data Simulation Based on Domain Knowledge

We have developed a laboratory testbed, consisting of a fault-enabled 24 VDC supply to power the three phases of the electromechanical actuator servo drives (ASD), and integrated the switch-mode power supply (SMPS) with a high-speed data acquisition (DAQ) unit (National Instruments USB-6251) [2], [28]. This enables us to leverage data from complex mechanical/electrical actuator systems including various components, utilizing direct experience and knowledge. This EMA system consists of four subsystems, each of which is composed of a variety of lower-level subsystems and components. Due to the tree structure of the system, a failure at the component level may propagate through the subsystem to the system and cause the failure of the system as a whole.

State-of-health (SOH) signals indicate the health conditions for all the components, subsystems, and the overall system. These data are generated with the guidance of prior information and domain knowledge. Without losing generality, there is a three-level system structure as referred to in Fig. 1. The first (top) level is the system level, which indicates the SOH of the overall system. The second level is the subsystem level where each node represents the SOH of a particular subsystem, such as a Logic SMPS (switch-mode power supply), a Motor SMPS, or an Actuator Servo Drive, etc. The third (bottom) level is the component level, and each node represents the SOH of a

particular component, such as capacitors, MOSFETs, etc. Prior probabilities and conditional probability tables are acquired from data and domain knowledge. In the validation of structure, learned CBN is compared with the true structure assumed at the beginning.

### B. Advanced Causal Structure Training and Results

In the FT, simple logic operations like AND, OR are used. In reality, there might be other unknown causes which the model fails to cover. In general BN, the undirected arcs might exist even with a deficiency of sensors collecting data in elements. In that case, we need experts making decisions to manually modify the learned structure to provide correct results in the validation step. To overcome aforementioned issues and provide better accurate causal structure, we have also developed advanced causal structure incorporating domain and engineering knowledge, such as FMEA.

There are cause-and-effect relationships for these nodes. For example, if failures such as package degradation happen in three phases, the transistor will malfunction and then the H-Bridge will fail. Therefore our advanced troubleshooting reasoning improved from [1], shown in Fig. 3, inherits information from both CBN and FT. In this case FT compensates CBN in describing the complex relationship among elements in the system. This demonstrates the capability and flexibility of our proposed solution in handling a more complex system by describing the complex relationships among elements of the four-layer system.



Fig. 3. Implementation of advanced troubleshooting reasoning

### C. Case Study

To demonstrate the proposed procedure of the reliability modeling of hierarchical systems, the EMA system is recomposed and investigated based on a hardware testbed with fault-enabled elements. Our testbed is capable of injecting controlled health status degradation and failure to the elements at different levels. Pulse width modulator (PWM) control under Motor SMPS has duplicated function with PWM control under ASD, and therefore is omitted for this case study. We characterize and represent the relationships of X31 (Motor SMPS), X21 (Capacitor) and X22 (MOSFET) from failure modes of EMA systems. For example, MOS transistors are used in the primary side of the circuit as a switch to control the

duration of primary current flow, and hence the output voltage, and they comprise the synchronous forward and catch rectifiers in the secondary side. At every switching instance, the MOSFET encounters a rapid change of voltage across its drain-source terminals ( $dv/dt$ ) as well as a spike of voltage kickback, due to the inductance of the circuit. At high values of  $dv/dt$ , the parasitic bipolar in the MOSFET can turn ON, causing uncontrolled operation. At higher voltages, the associated electric field is also very high, causing avalanche injection current and increasing the probability of turning the bipolar ON. This shoot-through current is a bypass current across the MOSFET through the gate capacitances. It results in giving rise to a current in the capacitance through the base resistance. Eventually, capacitor failures in the motor supply lead to an increased voltage ripple and reduction in current output. MOSFET failures lead to a complete loss of current and voltage. The BN representation of this investigated four-level system is shown in Fig. 4. The whole system,  $X_{4,1}$ , is composed of two subsystems,  $X_{3,1}$  and  $X_{3,2}$ , which consist of multiple components,  $X_{2,j}$ ,  $j = 1, \dots, 4$  and  $X_{1,j}$ ,  $j=1, \dots, 3$ . The conditional probabilities corresponding to the BN are summarized in Table I. For instance, the probability of  $X_{3,1}=0$  is 1 when  $X_{2,1}=0$  and  $X_{2,2}=0$ .

There are three types of information fused in this analysis. Table II summarizes the availabilities of components, subsystems and the overall system in terms of beta-prior distribution parameters. This type of prior information may be obtained from a compendium of various data sources for identical or similar components/subsystems under a similar utility environment. To be more specific, combined data sources provided include domain knowledge and judgments of expertise, test data of similar types of system elements, and historical studies on the same type of system elements. It is noticed that for prior parameters specification [10], the practitioners are encouraged to follow some existing work such as technical reports and IEEE standards. Because of the failure interdependency of the availability between components  $X_{2,1}$  and  $X_{2,2}$ , and that among  $X_{1,1}$ ,  $X_{1,2}$  and  $X_{1,3}$ , the conditional beta prior distributions of  $X_{2,1}$  and  $X_{1,2}$  are provided.

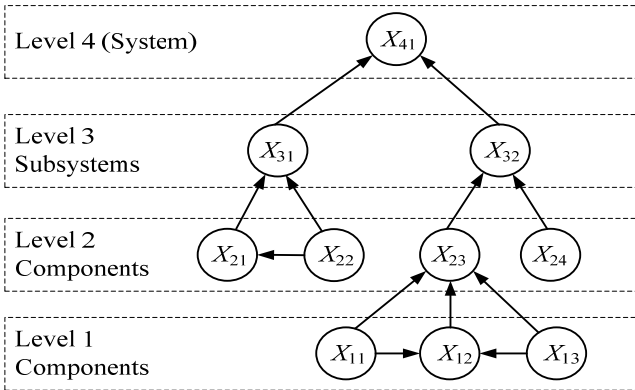


Fig. 4. BN representation of the case study system

TABLE I. CONDITIONAL PROBABILITY TABLE

$P(X_{23}=0)$	$X_{12}=0$	$X_{12}=1$
$X_{11}=0, X_{13}=0$	1	0
$X_{11}=1, X_{13}=0$	0	0
$X_{11}=0, X_{13}=1$	0	0
$X_{11}=1, X_{13}=1$	0	0
$P(X_{31}=0)$	$X_{21}=0$	$X_{21}=1$
$X_{22}=0$	1	0
$X_{22}=1$	0	0
$P(X_{32}=0)$	$X_{23}=0$	$X_{23}=1$
$X_{24}=0$	1	0
$X_{24}=1$	0	0
$P(X_{41}=0)$	$X_{31}=0$	$X_{31}=1$
$X_{32}=0$	1	1
$X_{32}=1$	1	0

TABLE II. BETA-PRIOR PARAMETERS

Component	$s_{(i,l,j),k}^0$	$n_{(i,l,j),k}^0$
$X_{12} X_{11}=0, X_{13}=0$	<b>803.2</b>	819.3
$X_{12} X_{11}=1, X_{13}=0$	<b>642.5</b>	688
$X_{12} X_{11}=0, X_{13}=1$	<b>548.3</b>	589.3
$X_{12} X_{11}=1, X_{13}=1$	<b>581</b>	723.5
$X_{11}$	<b>793.2</b>	823.1
$X_{13}$	<b>641.2</b>	664.3
Component	$s_{(i,l,j),k}^0$	$n_{(i,l,j),k}^0$
$X_{21} X_{22}=0$	<b>1121.4</b>	1164.5
$X_{21} X_{22}=1$	<b>878</b>	1013.6
$X_{22}$	<b>512.5</b>	537.2
Component	$s_{(i,l,j),k}^0$	$n_{(i,l,j),k}^0$
$X_{23}$	<b>231.5</b>	245.2
$X_{24}$	<b>189.3</b>	205.1
Subsystem	$s_{(i,l,j),k}^0$	$n_{(i,l,j),k}^0$
$X_{31}$	<b>80.1</b>	84.3
$X_{32}$	<b>58.6</b>	62.1
System	$s_{(i,l,j),k}^0$	$n_{(i,l,j),k}^0$
$X_{41}$	<b>43.1</b>	46.2

For component  $X_{23}$ , components  $X_{11}$ ,  $X_{12}$  and  $X_{13}$  are serially connected. Based on the given information about the components' prior distribution parameters, binomial test results and conditional probability in Tables I-III, the induced prior distribution for the availability of  $X_{23}$  is a  $beta(962, 135)$  distribution, according to Eq. (3). Similarly, the induced priors for the availability of subsystems  $X_{31}$  and  $X_{32}$  are  $beta(934, 95)$  and  $beta(422.6, 71.6)$  distributions, respectively. The native prior distribution parameters listed in Table II will be used to calculate the combined priors of  $X_{23}$ ,  $X_{31}$  and  $X_{32}$ . It is reasonable to assume that the native prior distribution parameter may be estimated based on prior data listed in Table II. Thus, the sum of the weighting coefficients  $w_1$  and  $w_2$  are set to be 1. In this study, we assign 0.75 and 0.25 to the induced prior and the native prior, respectively, i.e.,  $w_1 = 0.75$  and  $w_2 = 0.25$ . This indicates that more weight will be placed on the information aggregated from a lower level of the subsystem. As a result, the combined priors for  $X_{23}$ ,  $X_{31}$  and  $X_{32}$  are  $beta(414.875, 44.775)$ ,  $beta(720.775, 72.6)$  and  $beta(150.35, 21.3)$ , respectively. Posteriors for  $X_{23}$ ,  $X_{31}$  and  $X_{32}$  are obtained through Eq. (1) by integrating the combined priors and binomial test data. These posterior beta distributions are graphically shown as solid curves in Fig. 5, and their corresponding quantiles are summarized in Table IV.

The system is composed of two subsystems connected in parallel. The beta posterior parameters of subsystems are now treated as the “component” prior parameters. With the same aggregation method, the induced beta prior distribution for  $X_{41}$  is  $beta(2116.2, 19.8)$ . By assigning the same weighting coefficients, i.e.,  $w_1=0.75$  and  $w_2=0.25$ , the combined prior distribution for  $S$  is  $beta(562.125, 8.025)$ . Posterior is obtained through Eq. (1) by integrating the combined prior and its corresponding binomial test data.

TABLE III. BINOMIAL TEST RESULTS

Component	$S_{(i,l),k}$	$n_{(i,l),k}$
$X_{12} X_{11}=0, X_{13}=0$	252	265
$X_{12} X_{11}=1, X_{13}=0$	230	265
$X_{12} X_{11}=0, X_{13}=1$	234	265
$X_{12} X_{11}=1, X_{13}=1$	205	265
$X_{11}$	241	257
$X_{13}$	201	223
Component	$S_{(i,l),k}$	$n_{(i,l),k}$
$X_{21} X_{22}=0$	149	160
$X_{21} X_{22}=1$	135	160
$X_{22}$	161	170
Component	$S_{(i,l),k}$	$n_{(i,l),k}$
$X_{23}$	62	64
$X_{24}$	141	146
Subsystem	$S_{(i,l),k}$	$n_{(i,l),k}$
$X_{31}$	53	3
$X_{32}$	48	2
System	$S_{(i,l),k}$	$n_{(i,l),k}$
$X_{41}$	23	24

The system posterior is graphically shown in Fig. 5 and listed in Table IV, together with its corresponding quantiles. It is noted that in calculating the induced prior parameters for  $X_{21}$ ,  $X_{22}$  and  $X_{31}$ , the Kolmogorov-Smirnov (KS) [33]  $p$ -values are all larger than 0.05, indicating that the approximation is acceptable. Numerical values of posterior quantiles provide information regarding the reliabilities of the investigated system elements. For example, 0.97574 and 0.99203 are  $X_{41}$ 's 0.05 and 0.95 quantiles. They can be interpreted as based on the current available reliability information. The subjective belief of the reliability of the overall system is between [0.97574, 0.99203] with 90% credibility. The median of 0.98535 is the point estimate of the overall system reliability. Maintenance decisions can be made according to such quantities.

The impacts of components' failure interdependency are also investigated in this case study. The BN shows that components  $X_{21}$  and  $X_{22}$ , are not independent, and the availability of  $X_{22}$  has a positive impact on that of  $X_{21}$ . When  $X_{22}$  is failed,  $X_{21}$  is more likely to fail. This can be seen from the binomial test data in Table III and prior parameters in Table II. The same failure interdependency exists among  $X_{11}$ ,  $X_{12}$  and  $X_{13}$ . If the method in

[27] is adopted and the failure interdependencies between components are neglected, the predicted mean of availability of serial subsystem  $X_{21}$  and  $X_{22}$  will be higher than their true values, shown as the dashed curves in Fig. 5. However, this does not indicate that we should neglect the interdependencies, since if the interdependent relationship is strong, the discrepancy will be obviously large. Moreover, the value of the proposed methodology is to quantify the reliability of the system, which includes interdependent subsystems/components. Such interdependencies are often neglected in much existing research by assuming independent subsystems/components.

One concern related to the problem scale for large systems is the quality of the prior approximation, i.e., the approximation of the induced priors. To verify the approximation results, KS test is applied to test whether the approximation quality is acceptable. Table IV lists the  $p$ -values of K-S tests. Since all  $p$ -values are larger than significance level of 0.05, they imply that the approximation quality is satisfactory.

## VI. DISCUSSION ON APPLICABILITY

Expert troubleshooting and decision support systems are needed to achieve significant savings, which is accomplished by learning what diagnostic actions lead to correct outcomes and minimizing wasted efforts while achieving reduced maintenance costs. According to an analysis [34] by NAVAIR, it is possible to achieve 20% maintenance cost savings on deployed electronic systems. The presented approach will be used to develop an expert troubleshooting tool as a part of an integrated system health management (ISHM) system for complex mechanical/electrical system faults/failures. It will ultimately provide substantial benefits to the maintainer/operator of complex systems and maintenance facilities.

The improved reliability-based decision support methodology presented in this paper provides a foundation for an effective troubleshooting and decision support system to the maintainer/operator of complex systems and maintenance facilities. Integration of the CBN into condition-based maintenance (CBM) systems will enable maintainers to interpret complex interactions among components and subsystems embedded within the system, facilitating root cause analysis and reducing maintenance costs. As an illustration of how this can be done, consider integration with a ground-based application that provides an easy-to-understand fusion of parameters and required maintenance, so maintenance personnel can have an extra angle of insight into the systems for which they are responsible.

TABLE IV. RESULTS OF THE AVAILABILITY ANALYSIS FOR  $X_{23}$ ,  $X_{31}$ ,  $X_{32}$ , AND  $X_{41}$ 

Component / Subsystem / System	Induced Prior	KS test $p$ -value	Beta Posterior	Posterior Quantiles		
				0.05	0.5	0.95
$X_{23}$	$\beta(962,135)$	0.759	$\beta(476.575, 46.775)$	0.88927	0.91115	0.93019
$X_{31}$	$\beta(934, 95)$	0.954	$\beta(773.775, 75.6)$	0.89440	0.91132	0.92649
$X_{32}$	$\beta(422.6, 71.6)$	0.859	$\beta(198.35, 23.3)$	0.85915	0.89607	0.92655
$X_{41}$	$\beta(2116.2, 19.8)$	0.969	$\beta(585.125, 9.025)$	0.97574	0.98535	0.99203



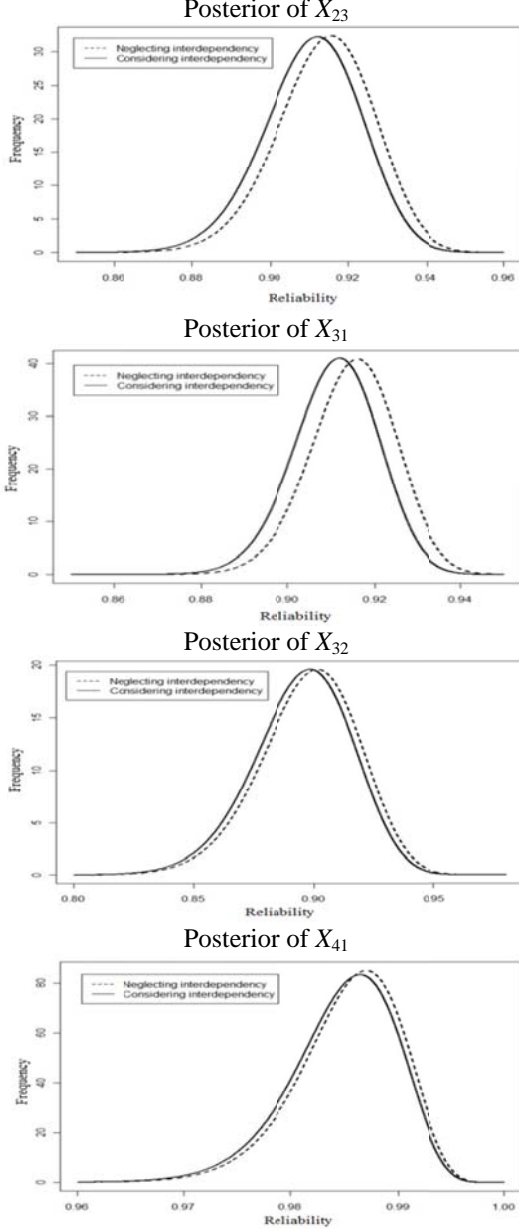


Fig. 5. Posterior distributions of component, subsystem and system reliability

## VII. CONCLUSION

The proposed advanced causal structure and reliability modeling of multilevel hierarchical systems are used to effectively represent the hierarchical structure of systems and the failure interdependency among elements at different levels with FMEA and test data acquired from complex EMA systems, integrating the SMPS with a high-speed DAQ unit.

We have developed advanced troubleshooting reasoning with causal analysis incorporating domain and engineering knowledge, and explained the importance. In our approach, causal analysis allows us to represent the component interactions and cascaded failure/degradation propagation. Based on the BN representation, a generic Bayesian

framework is developed to aggregate the multilevel information with binomial test data and prior knowledge that is modeled with beta distribution. The case study based on the data collected from the testbed shows the effectiveness of the proposed methodology. Such numerical values can help maintenance personnel make decisions such as whether maintenance tasks should be assigned.

Our approach presented in this paper provides a foundation for an effective decision support system to the maintainer of complex systems and maintenance facilities. Integration of our approach into condition-based maintenance (CBM) systems will enable maintainers to interpret complex interactions among components and subsystems embedded within the system, facilitating root cause analysis and reduce maintenance costs.

We expect the approach described in this paper to become a very important decision support methodology that will enable enhancements in flight safety and CBM by increasing availability and mission-effectiveness while reducing maintenance costs.

## APPENDIX

Denote the approximate induced prior for  $X_{l,j}$  by  $P(X_{l,j}=0) \sim \text{beta}(b,c)$ . Then, the first two moments of this distribution are

$$\tilde{M}_1 = \frac{b}{b+c}, \text{ and } \tilde{M}_2 = \frac{b(b+1)}{(b+c)(b+c+1)}, \quad (7)$$

respectively. Here, “ $\sim$ ” denotes these two moments are derived from the approximate distribution. Next, we derive the first two moments of the exact distribution for  $X_{l,j}$ , denoted by  $M_1$  and  $M_2$ , respectively. The exact distribution is in the form specified by plugging (3) into (2), i.e.,

$$P(X_{l,j} = 0) = \sum_h \alpha_{(l,j)h} \prod_i P(PA_i(X_{l,j}) = \kappa | \mathbf{PA}(PA_i(X_{l,j})) = \mathbf{k})). \quad (8)$$

Recall that  $h$  is used to index each combination of values for the parents of  $X_{l,j}$ ,  $\kappa$  is the value for  $PA_i(X_{l,j})$  in  $h$ , and  $\mathbf{k}$  is a vector of the values for  $\mathbf{PA}(PA_i(X_{l,j}))$  in  $h$ , and  $l=0,1$ . To facilitate subsequent derivation, (8) can be further written as:

$$P(X_{l,j} = 0) = \sum_h \alpha_{(l,j)h} \prod_i \left\{ \kappa - P(PA_i(X_{l,j}) = 0 | \mathbf{PA}(PA_i(X_{l,j})) = \mathbf{k})) \right\} \quad (9)$$

The first moment of  $P(X_{l,j}=0)$  is the mean, i.e.,

$$\begin{aligned} M_1 &= E(P(X_{l,j} = 0)) \\ &= \sum_h \alpha_{(l,j)h} \prod_i \left\{ \kappa - E \left[ P(PA_i(X_{l,j}) = 0 | \mathbf{PA}(PA_i(X_{l,j})) = \mathbf{k})) \right] \right\} \\ &= \sum_h \alpha_{(l,j)h} \prod_i \left\{ \kappa - \frac{s_{(i,l,j),k} + s_{(i,l,j),k}^0 + 1}{n_{(i,l,j),k} + n_{(i,l,j),k}^0 + 2} \right\}. \end{aligned} \quad (10)$$

The second moment of  $P(X_{l,j}=0)$  is the mean of  $P(X_{l,j}=0)^2$ , i.e.,

$$\begin{aligned} M_2 &= E(P(X_{l,j} = 0))^2 \\ &= \sum_h \alpha_{(l,j)h} \prod_i \left\{ \kappa^2 - 2\kappa \frac{s_{(i,l,j),k} + s_{(i,l,j),k}^0 + 1}{n_{(i,l,j),k} + n_{(i,l,j),k}^0 + 2} + \right. \end{aligned}$$

$$\left. \frac{(s_{(i,l,j),k} + s_{(i,l,j),k}^0 + 1)(s_{(i,l,j),k} + s_{(i,l,j),k}^0 + 2)}{(n_{(i,l,j),k} + n_{(i,l,j),k}^0 + 2)(n_{(i,l,j),k} + n_{(i,l,j),k}^0 + 3)} \right\}. \quad (11)$$

Then, for a beta distribution, the parameters  $b$  and  $c$  can be defined as

$$\begin{aligned} b &= M_1(M_1 - M_2) / (M_2 - M_1^2), \\ c &= (1 - M_1)(M_1 - M_2) / (M_2 - M_1^2). \end{aligned} \quad (12)$$

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